



**Quantum Monte Carlo study of
straintronic response of 2D materials:
monolayer phosphorene and MoS₂**

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SLOVAK ACADEMY OF SCIENCES



Funding:

- **H2020**
- **APVV-21-0272**
- **VEGA 2/0070/21**
- **PRACE**
- **EuroHPC**



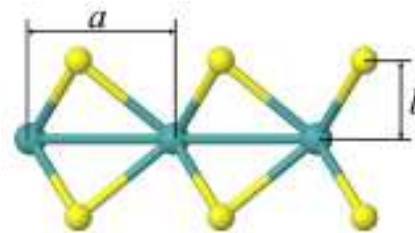
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project no.
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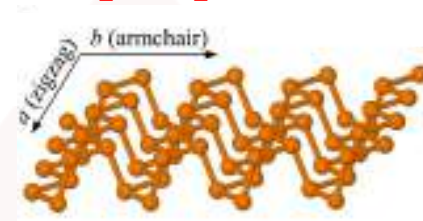
Intro. 2D materials

quintessential
straintronic material

MoS₂

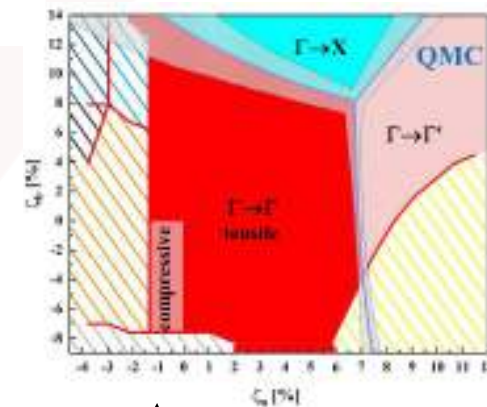
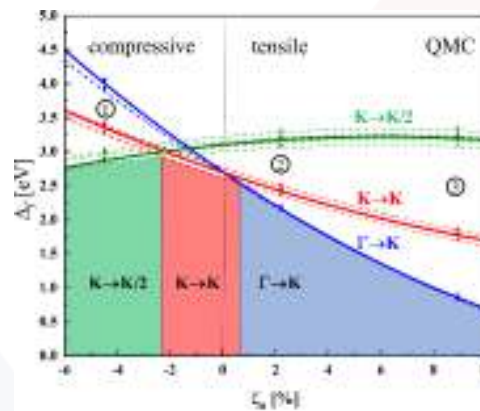


phosphorene



- Calculations: 1) band gap
- 2) strain effects
- 3) deformation energy

„phase diagrams“ of excitations



Summary

□ a number of 2D materials prepared:

- **exfoliation**
- **epitaxial growth**



Nobel prize for physics (2010)
Geim & Novoselov

□ electronic properties:

metals \Leftrightarrow semimetal \Leftrightarrow **semiconductor** \Leftrightarrow **wide gap insulator**

Pd, Rh graphene

TMD
phosphorene

h-BN

□ high carrier mobility

□ often direct gap

2D materials highly sensitive to **strain**

2D materials can **sustain strains** well in excess of **10%**: graphene $\approx 25\%$ and $\text{MoS}_2 \approx 11\%$

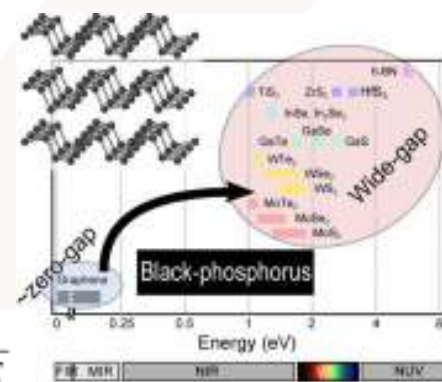
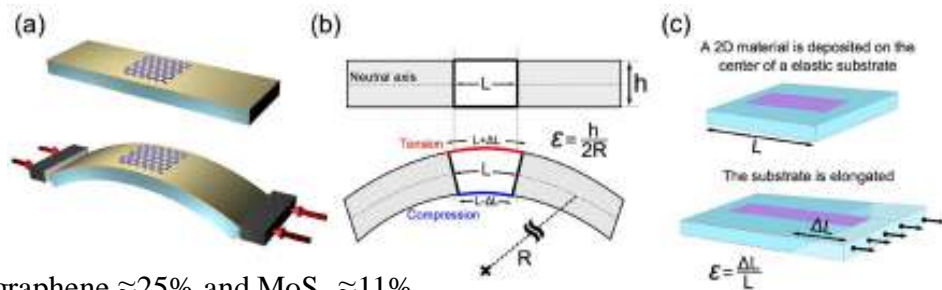
R. Roldán, A. Castellanos-Gomez, E. Capelluti, and F. Guinea, *J. Phys.: Condens. Matter* **27**, 313201 (2015)

tuning properties via **strain - band gap** for given applications

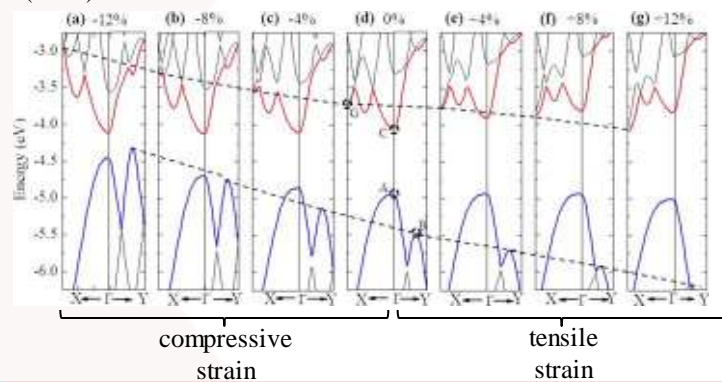
Castellanos-Gomez. A., *J. Phys. Chem. Lett.* **6**, 4280 (2015)

- **effective mass** \Rightarrow **carrier mobility**

Y. S., Y. Chen, and C. Jiang, *InfoMat.* **3**, 397420 (2021)



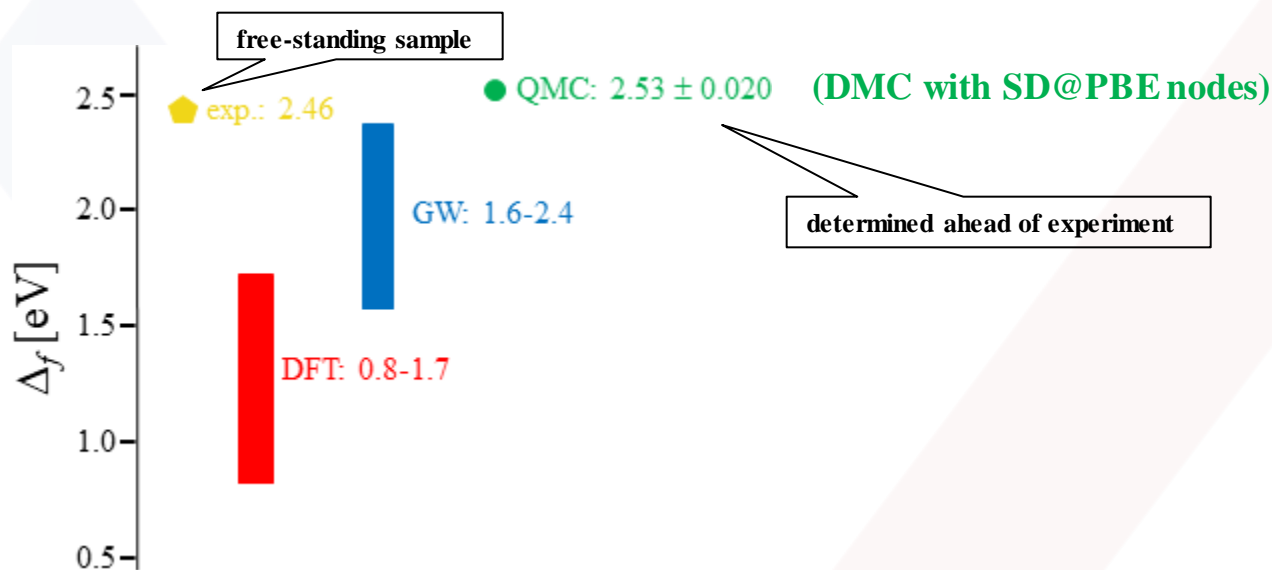
$$m^* = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}$$



2D materials: phosphorene gap

accuracy: quasiparticle band gap

example: monolayer phosphorene



E. Gaufrès, F. Fossard, V. Gosselin, L. Sponza, F. Ducastelle, Z. Li, S. G. Louie, R. Martel, M. Côté, and A. Loiseau, *Nano Lett.* 19, 8303 (2019).
 T. Frank, R. Derian, K. Tokar, L. Mitas, J. Fabian, and I. Stich, *Phys. Rev. X* 9, 011018 (2019).

how can the band gap be calculated?

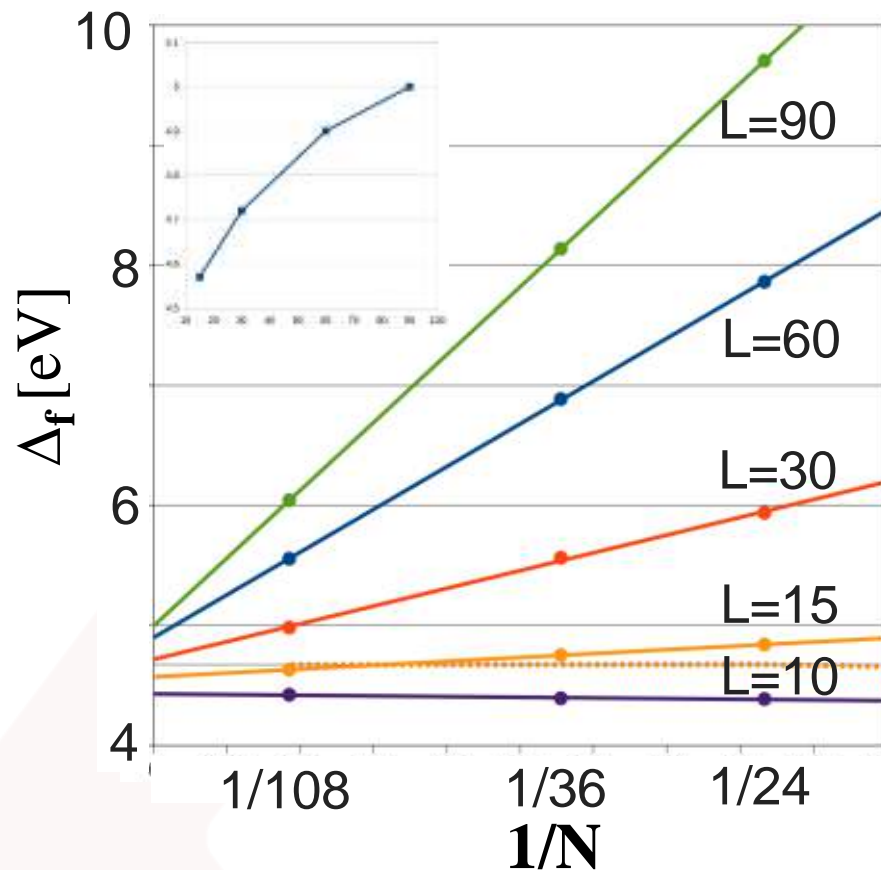
- quasiparticle band gap

$$\Delta_f = (E_{N_{e+1}}^{\ominus} - E_{N_e}) - (E_{N_e} - E_{N_{e-1}}^{\oplus})$$

how to calculate for a 2D crystal???

2D crystal is in reality 2D+h system

???how to apply
neutralizing background ???

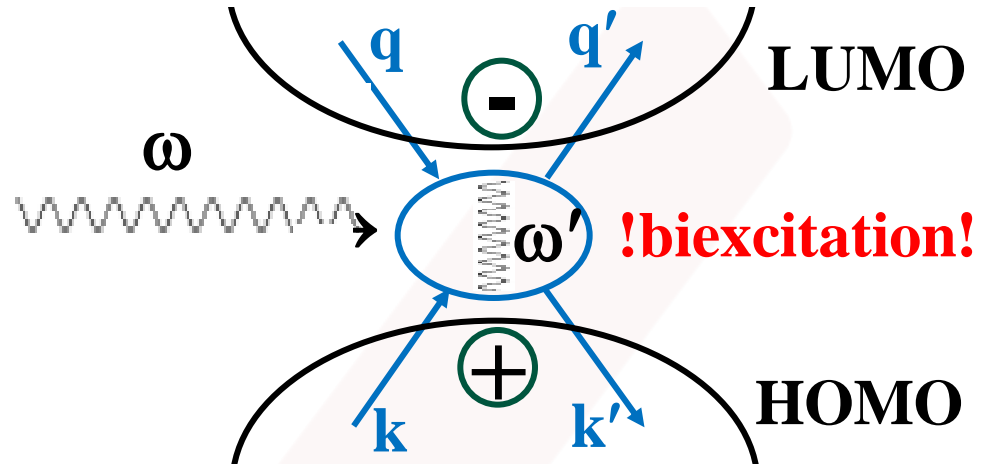
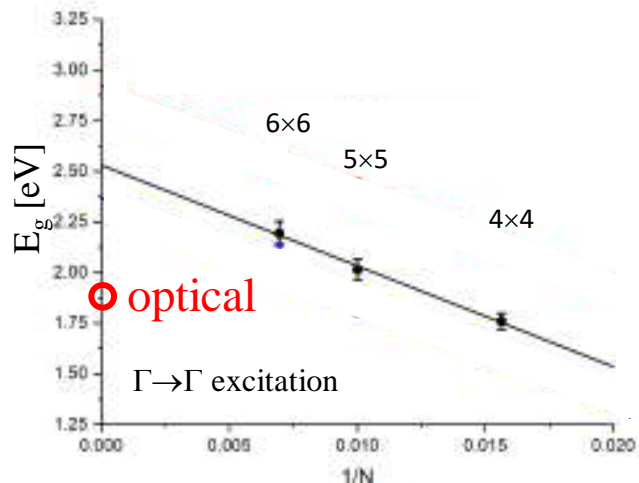


how can the band gap be calculated?

- excitation/neutral band gap

$$\Delta_e \approx E_v^s = E_1^s - E_0^s$$

what does that correspond to:
optical **quasiparticle**

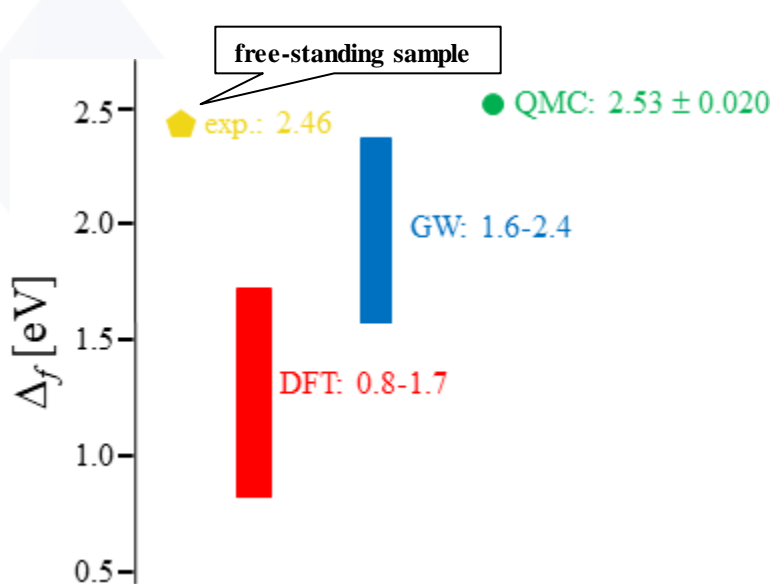


⇒ no exciton possible!!!

- in 2D size larger than supercell
- e-h pair cannot be formed from single-particle Bloch states
- DMC gaps@PBE nodes
gaps@hybrids equal

accuracy: quasiparticle band gap

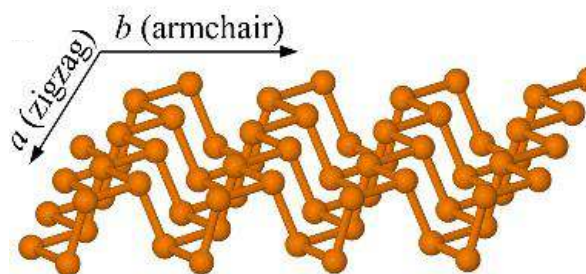
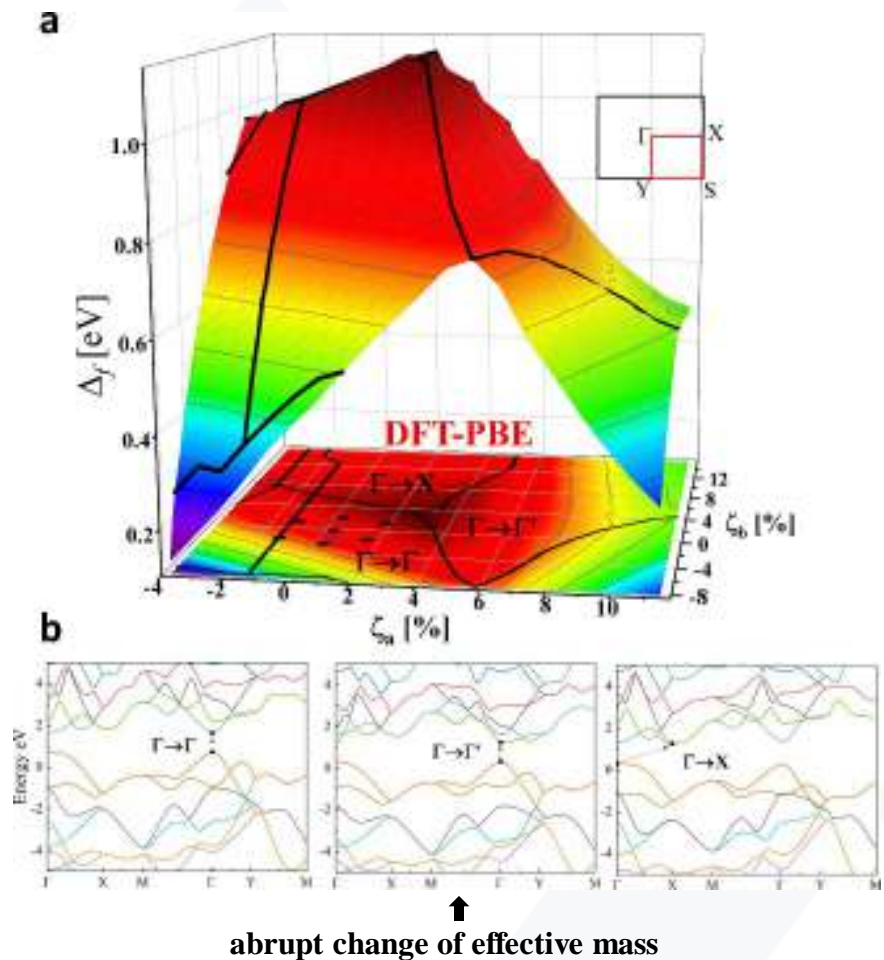
example: monolayer phosphorene



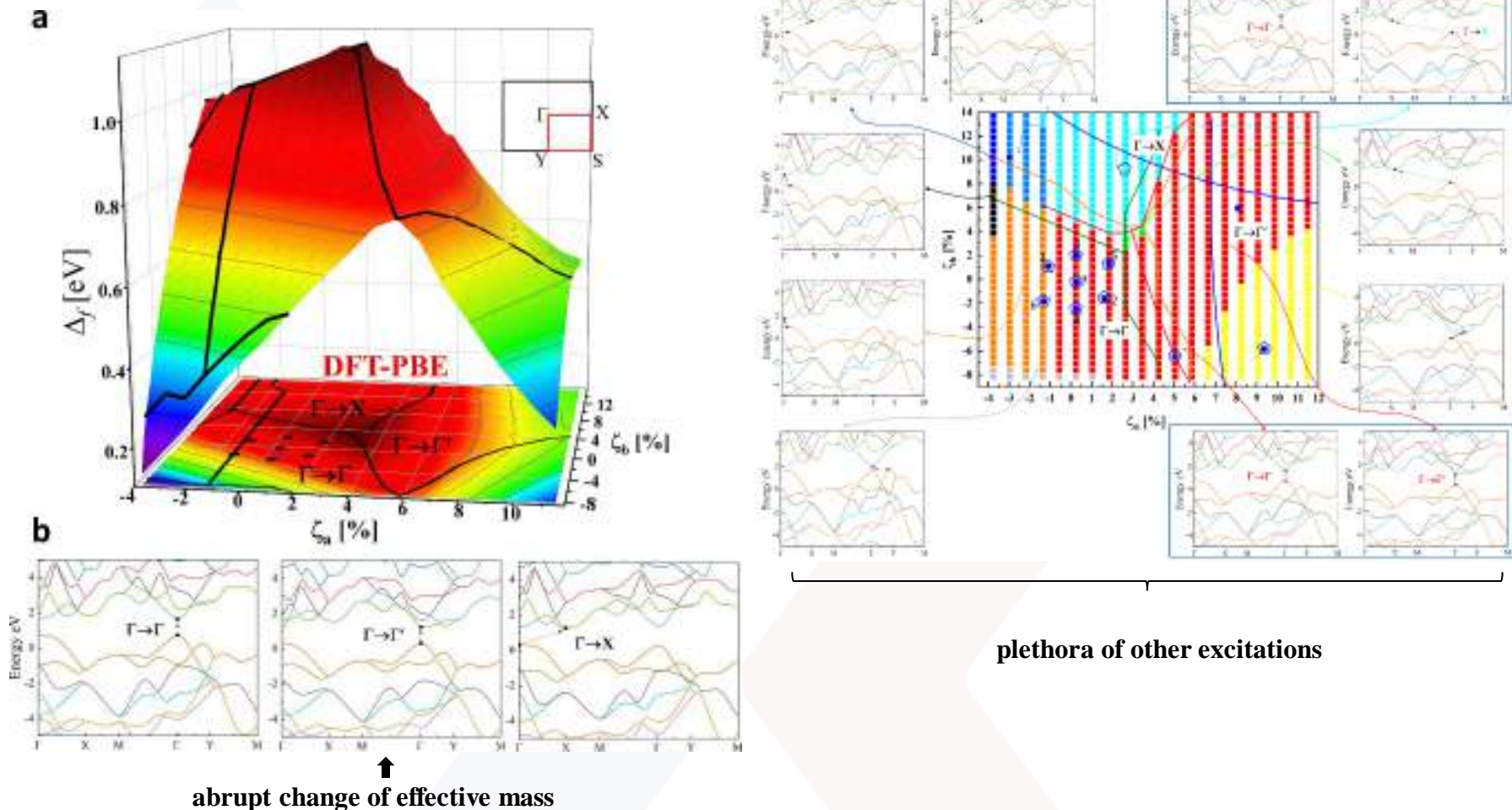
need accuracy better than
 \Rightarrow DFT & GW: quasiparticle gap \Rightarrow QMC
 GW-BSE: for excitons
 \Rightarrow under **strain bias** bound to **increase**
 theory & experiment

E. Gaufrès, F. Fossard, V. Gosselin, L. Sponza, F. Ducastelle, Z. Li, S. G. Louie, R. Martel, M. Côté, and A. Loiseau, Nano Lett. 19, 8303 (2019).
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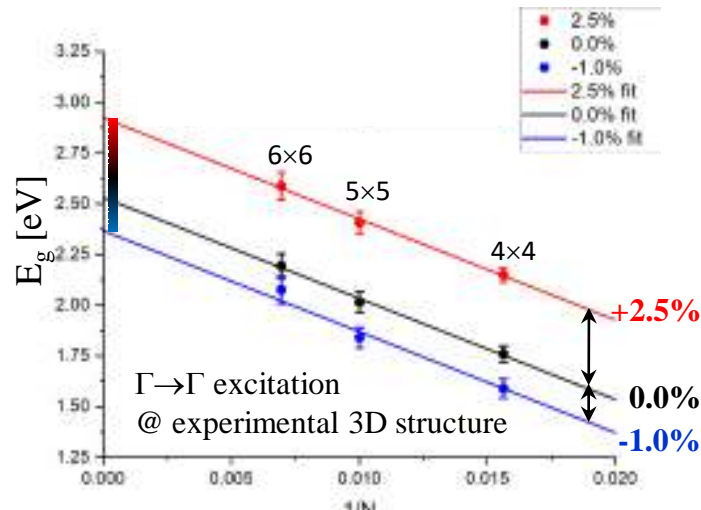
qualitative picture: pre-screened by DFT-PBE DFT



qualitative picture: pre-screened by DFT-PBE DFT



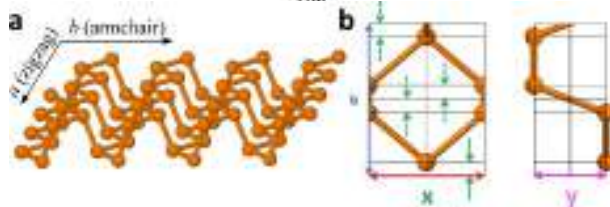
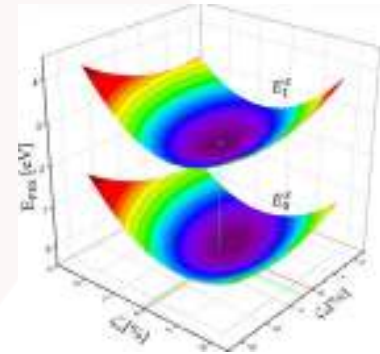
quantitative picture: FNQMC (SD with DFT-PBE nodes)



- the strain band gap **tuning** is quite **strong**
- quasiparticle band gap **scales linearly** with the applied **strain**
- **highly nontrivial** as the band gaps are differences between ground- and excited-state PESs.

converged ground- and excited-state **PESs**
parallel; only vertical & lateral offsets

diagonal

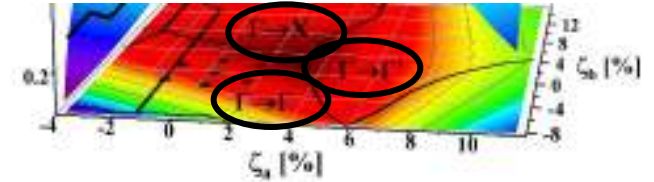


\Rightarrow 4 parameters **a**, **b**, **x**, and **y**

bracketted by at least 25 QMC points

fixing all 4 parameters **a**, **b**, **x**, and **y**: **quadrivariate paraboloid function** $E_0(a, b, x, y)$, minimize w.r.t. **x**, and **y** for each $(a, b) \Rightarrow E_0(a, b)$
excited state E_1 only computed at the minimum w.r.t. **x**, and **y** $\Rightarrow E_1(a, b)$

structural approximants & finite size scaling:
7 points bracketing the minimum



$\Gamma \rightarrow \Gamma / \Gamma \rightarrow \Gamma'$:

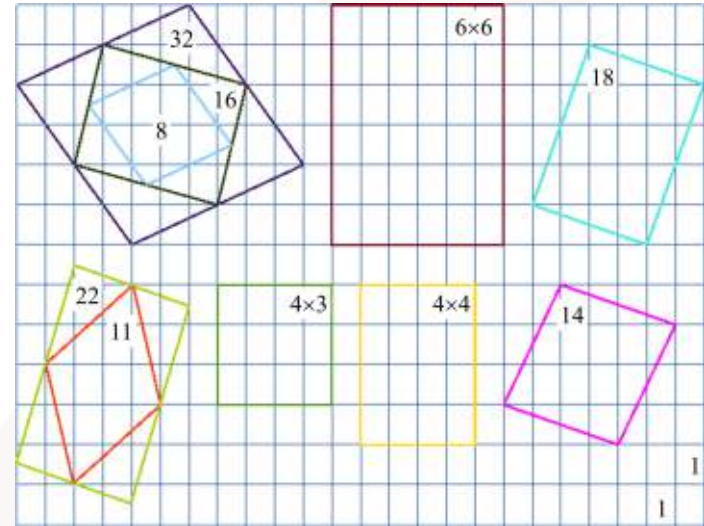
approximants:

- 11 prim. u.c. | 4D**
- 16 prim. u.c. | 4D**
- 18 prim. u.c. | 4D**
- 22 prim. u.c. | 4D**

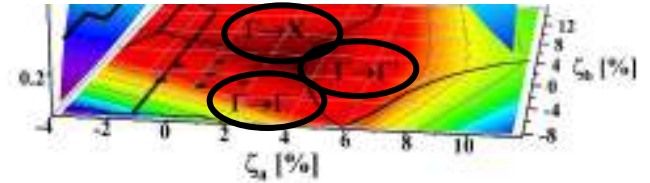
$\Gamma \rightarrow X$:

approximants:

- 14 prim. u.c. | 4D**
- 16 prim. u.c. | 4D**
- 18 prim. u.c. | 4D**
- 22 prim. u.c. | 4D**



structural approximants & finite size scaling:
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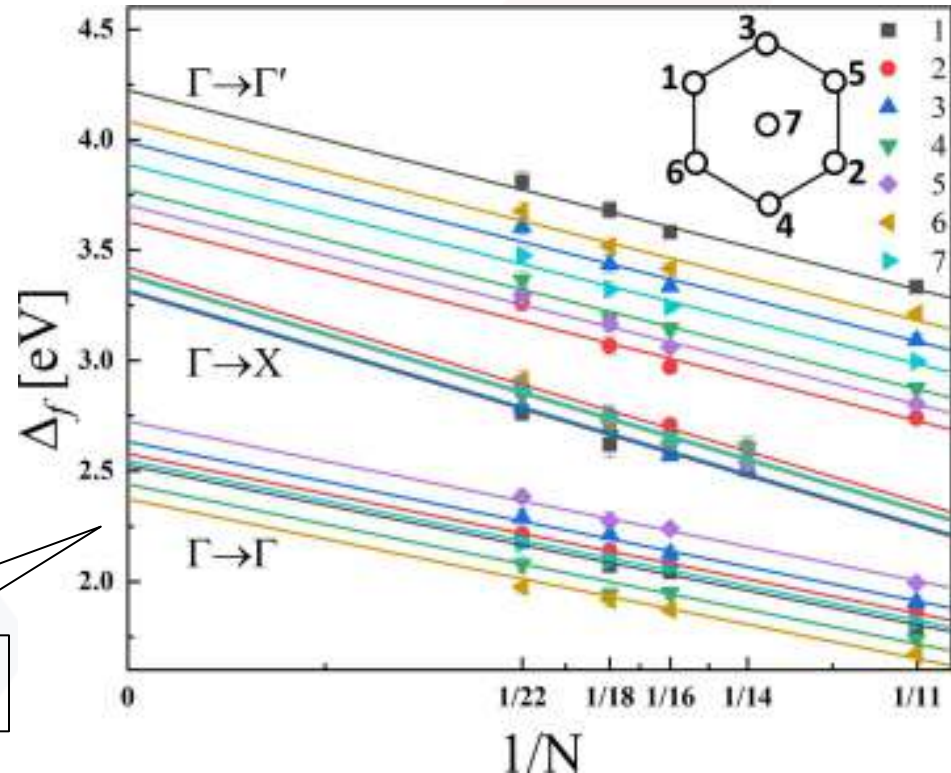
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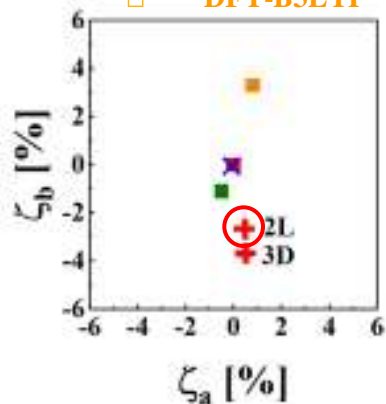
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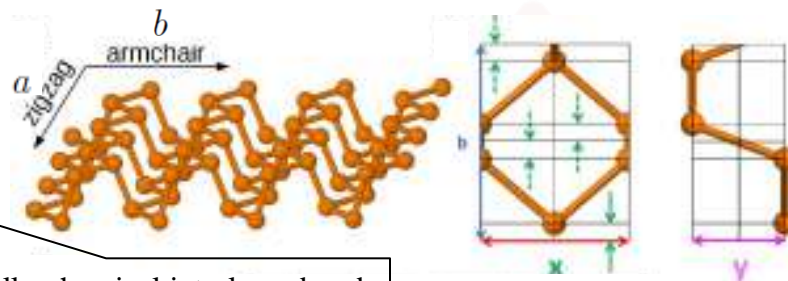
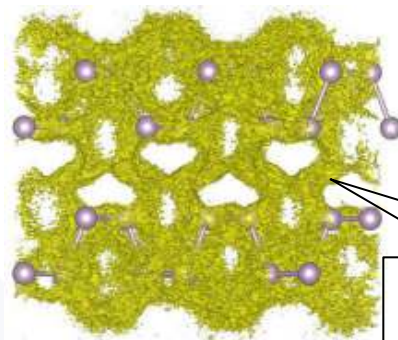
!no twist averaging possible!



- × QMC
- DFT-PBE
- DFT-HSE
- DFT-B3LYP



2-layer phosphorene



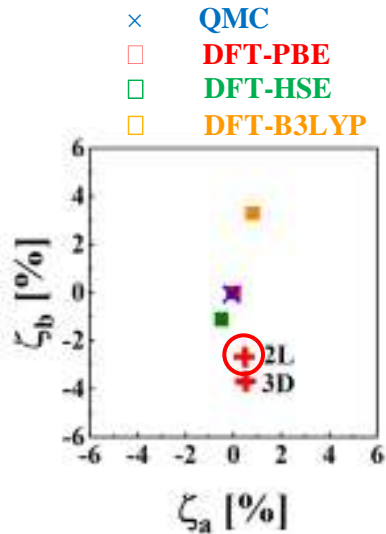
partially chemical interlayer bond
(between lone pairs on P's)

expt.

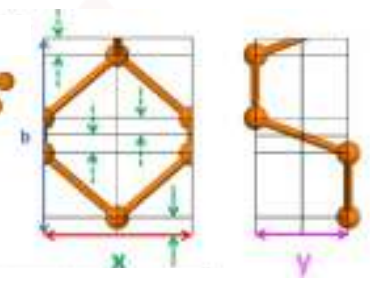
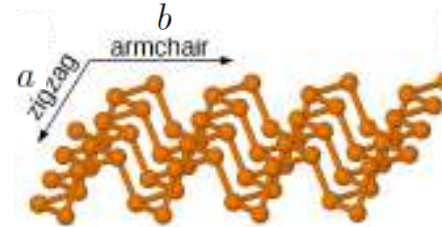
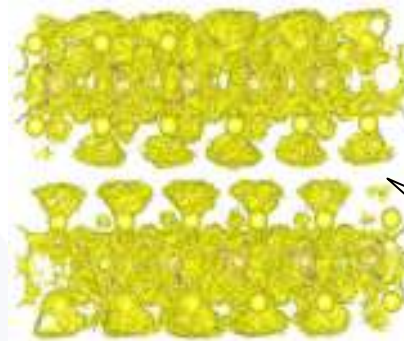
	QMC 11-cell	QMC 16-cell	QMC 22-cell	DFT-PBE	3D-BP
ground state					
<i>a</i>	6.229±0.008	6.238±0.002	6.230±0.002	6.235	6.2618
<i>b</i>	8.651±0.017	8.688±0.008	8.707±0.006	8.711	8.2700
<i>x</i>	0.748±0.005	0.759±0.002	0.765±0.001	0.770	0.6367
<i>y</i>	3.987±0.007	3.978±0.003	3.985±0.002	3.975	4.0280
E_0^g	-716.563±0.002	-716.498±0.0008	-716.469±0.0007	-717.711	NA
excited state					
<i>a</i>	6.227±0.027	6.222±0.003	6.224±0.003	6.222	NA
<i>b</i>	8.515±0.178	8.607±0.012	8.664±0.010	8.543	NA
<i>x</i>	0.728±0.044	0.747±0.003	0.761±0.002	0.739	NA
<i>y</i>	4.053±0.018	4.031±0.002	4.020±0.002	4.051	NA
E_1^g	-716.415±0.005	-716.375±0.0007	-716.375±0.001	-717.643	NA

$$\begin{aligned}
 a &= 3.30 \pm 0.003 \text{ \AA} \\
 b &= 4.61 \pm 0.006 \text{ \AA} \\
 x &= 0.405 \pm 0.001 \text{ \AA} \\
 y &= 2.109 \pm 0.001 \text{ \AA}
 \end{aligned}$$

not known experimentally



2-layer MoS₂



pure vdW interaction

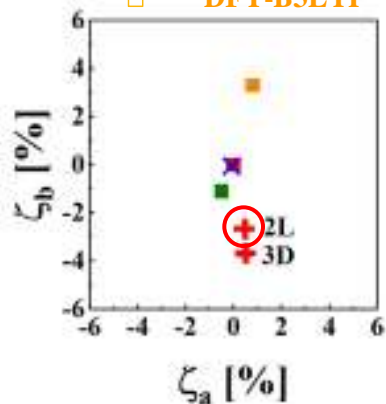
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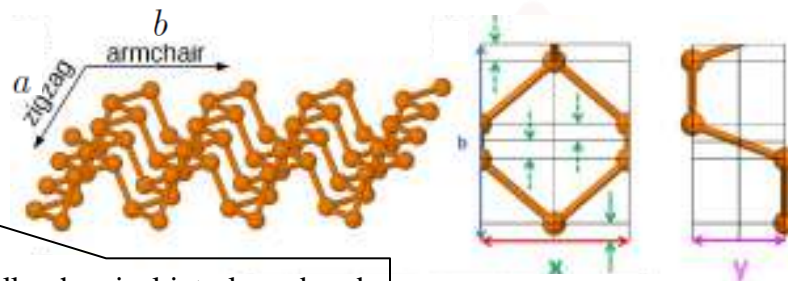
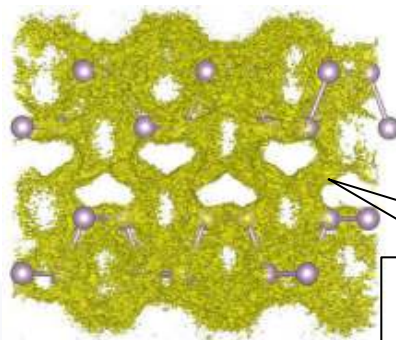
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not known experimentally

$$\Delta_f = 2.53 \pm 0.020 \text{ eV} \Leftrightarrow 2.46 \text{ eV}$$

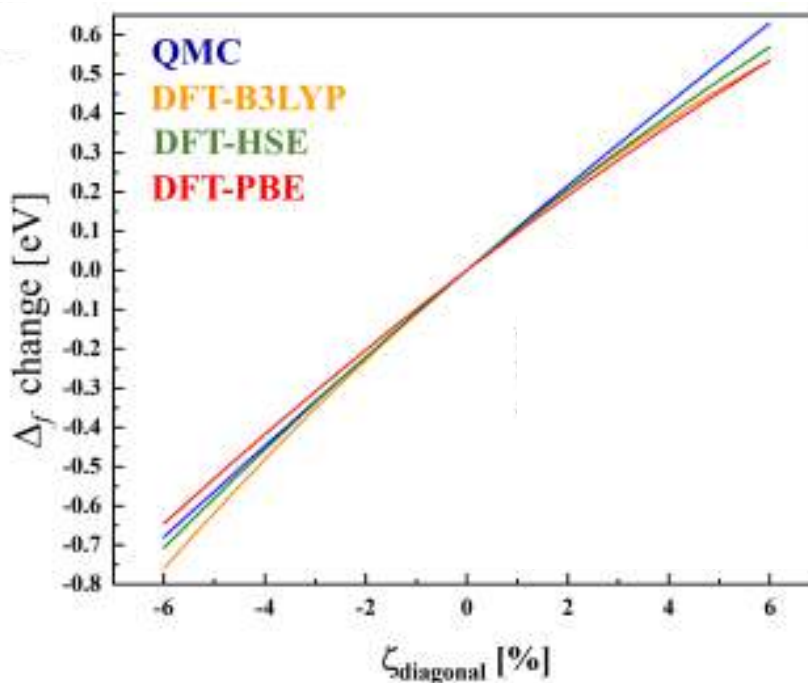
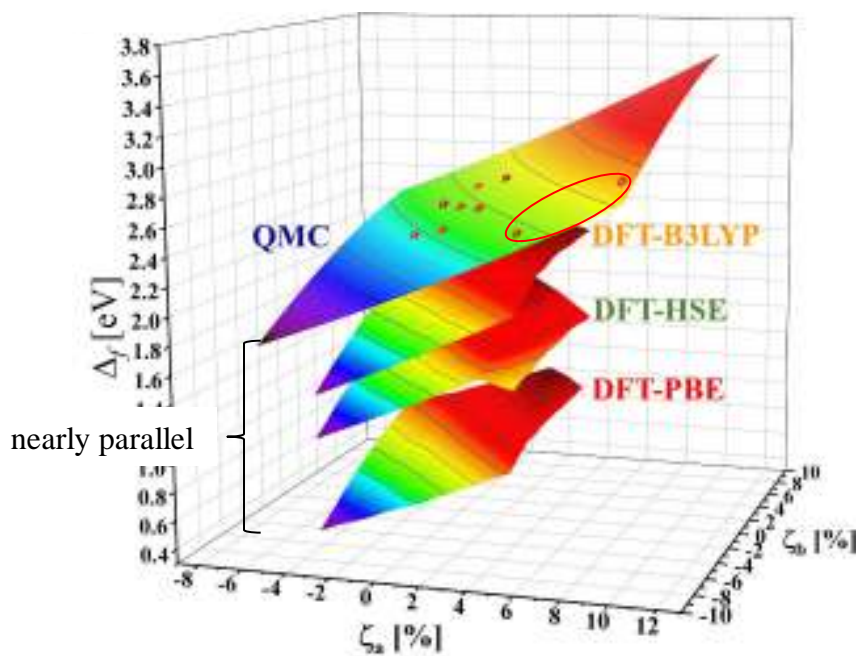
experimental value for freestanding phosphorene

↑
(almost) **chemical precision**
(no vibronic and zero-point corrections)

band gap

$\Gamma \rightarrow \Gamma$ excitation

gauge factor



QMC consistently gives **larger gap** than any common DFT functional (**PBE**, **HSE**, **B3LYP**),
 ! $\zeta = 0$ agrees with experiment!

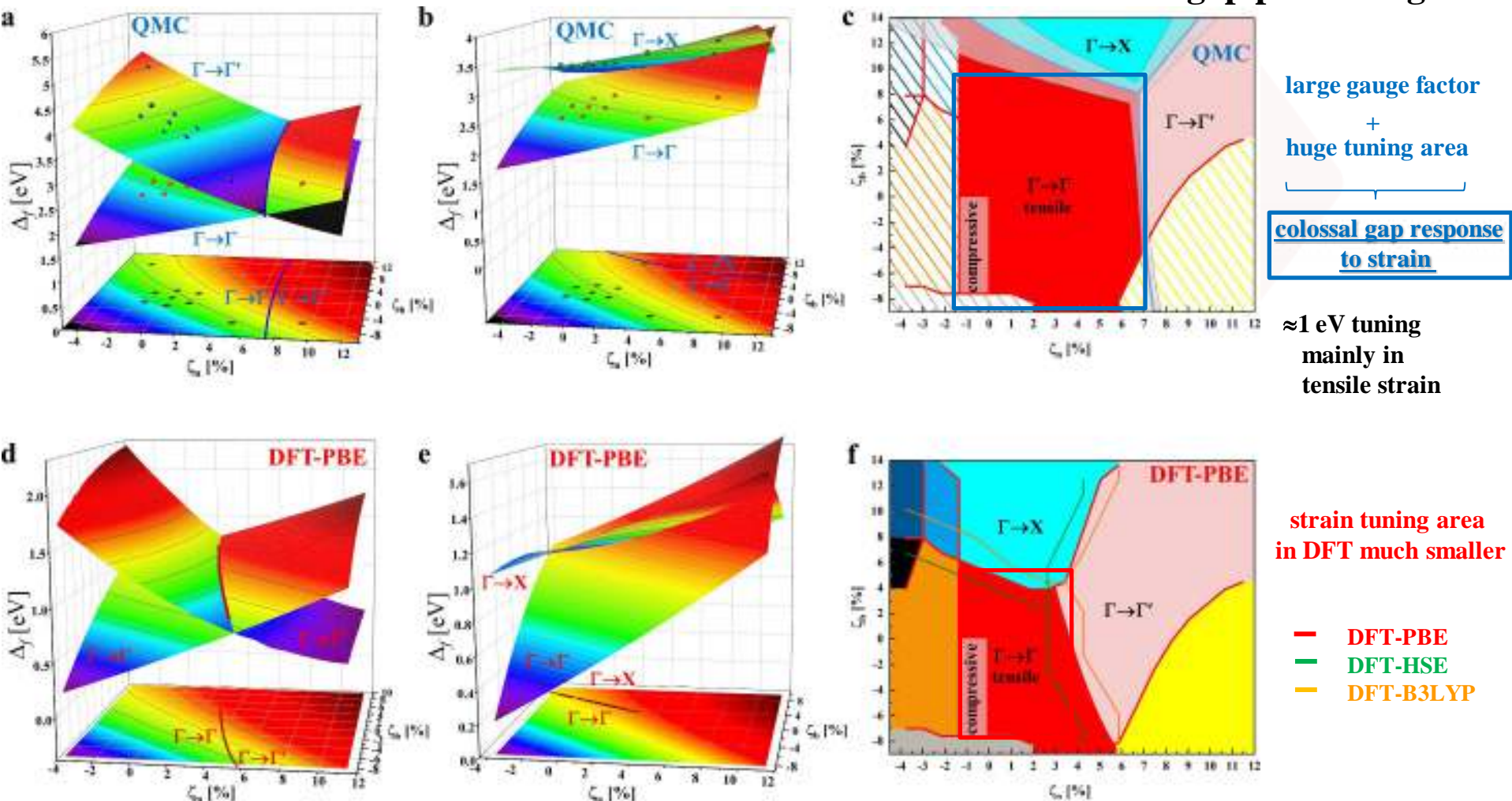
gauge factor (change of Δ_Γ) very **similar to DFT**
 (if computed against the respective minimum)

large gauge factor: ≈ 100 meV/%

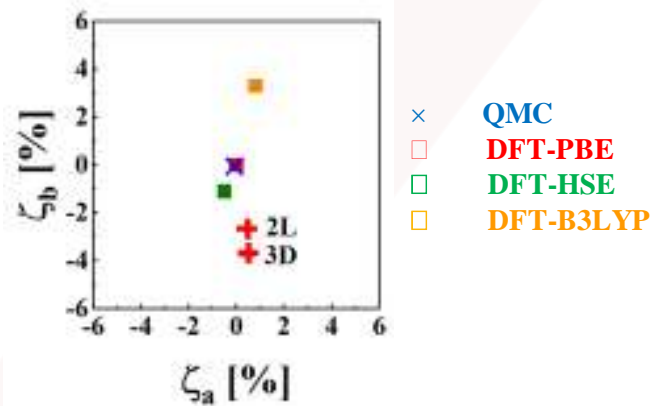
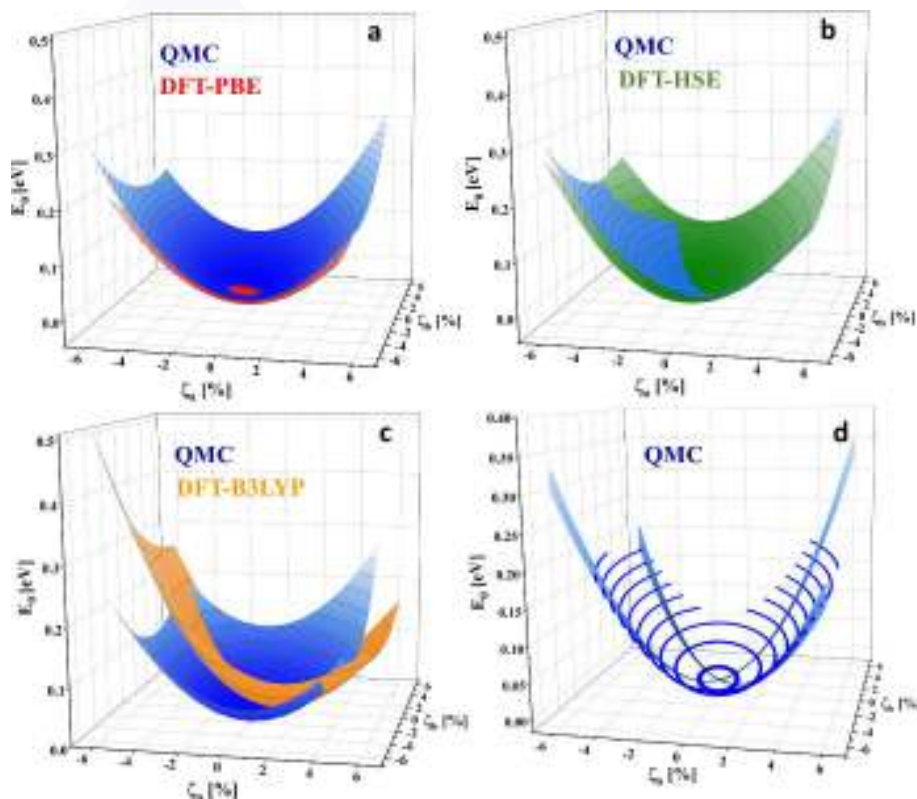


main straintronic materials **MoS₂: $64 \pm 5 / 68 \pm 5$ meV/%**

quantitative picture: QMC $\Gamma \rightarrow \Gamma$ $\Gamma \rightarrow \Gamma'$ $\Gamma \rightarrow X$ boundaries \Rightarrow band gap phase diagrams

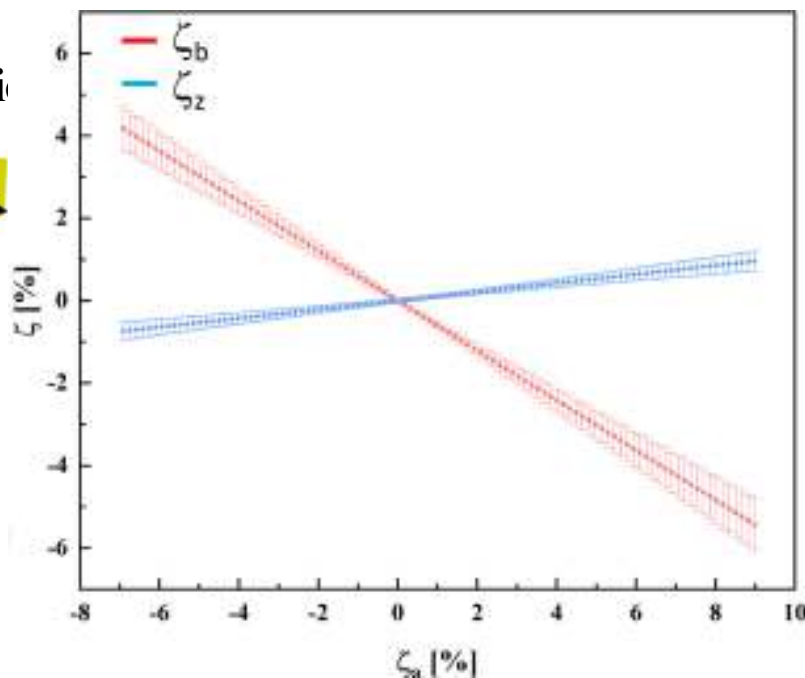
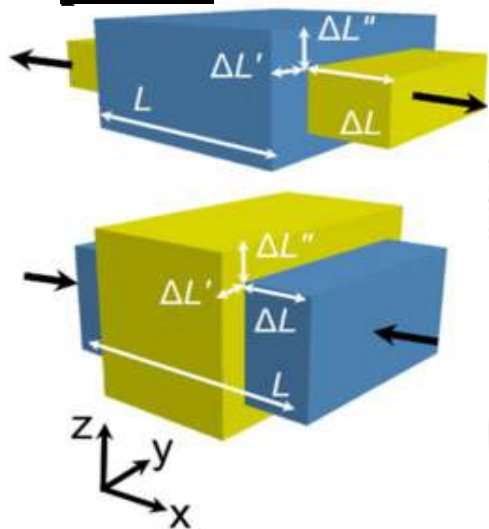


Y. Huang, A. Faizan, M. Manzoor, J. Brndiar, L. Mitás, J. Fabian, and I. Štich, Phys. Rev. Reserach 5, 033223 (2023).



Poisson's ratio

positive Poisson's ratio

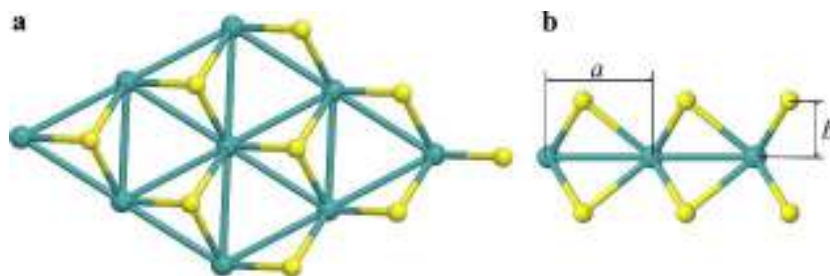


positive in-plane

negative out-of-plane

Poisson's ratio

auxetic material



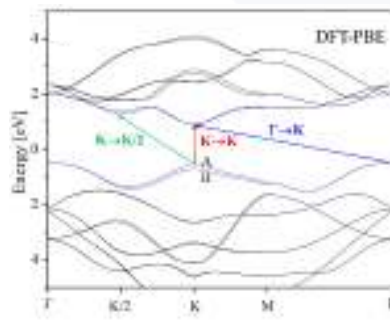
consider only diagonal strain (**a**)

⇒ 2 parameters **a**, **b**

fixing 2 parameters **a**, **b**: **bivariate paraboloid function** $E_0(\mathbf{a}, \mathbf{b})$, minimize w.r.t. **b** for each **a** ⇒ $E_0(\mathbf{a})$

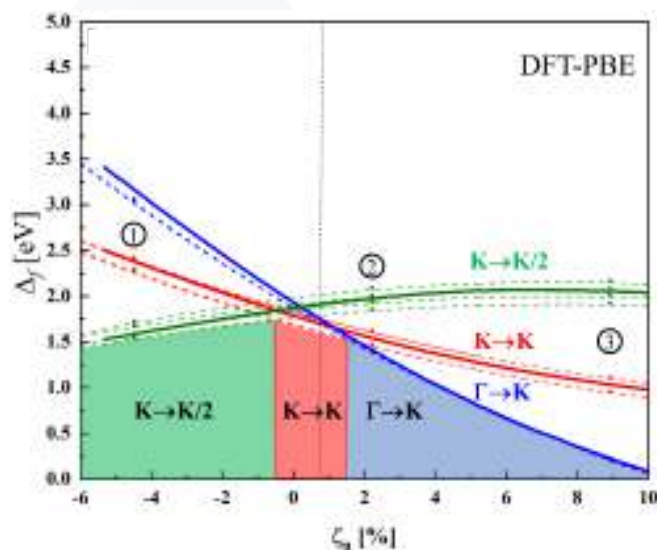
excited state E_1 only computed at the minimum w.r.t. **b** ⇒ $E_1(\mathbf{a})$

spin-orbit couplings (SOC) small but non-negligible ⇒ **SOC** added a posteriori **perturbatively** based on **DFT-PBE**

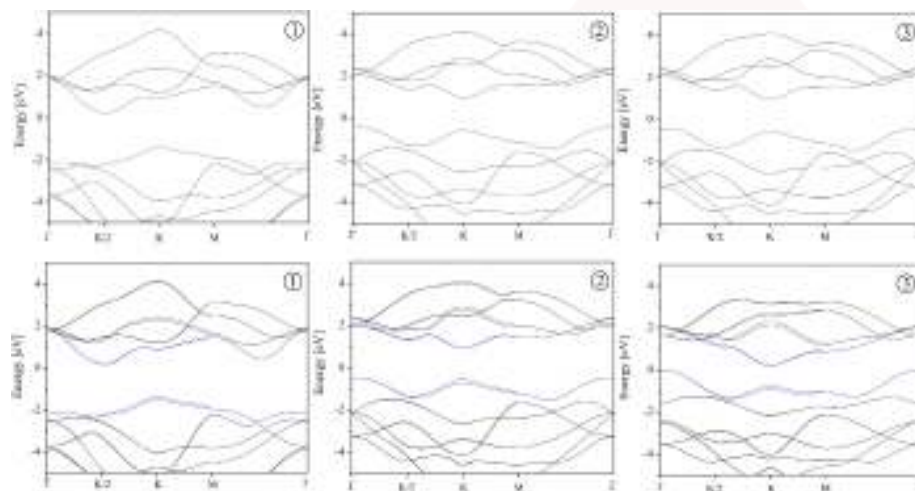


qualitative picture: pre-screened by DFT-PBE DFT

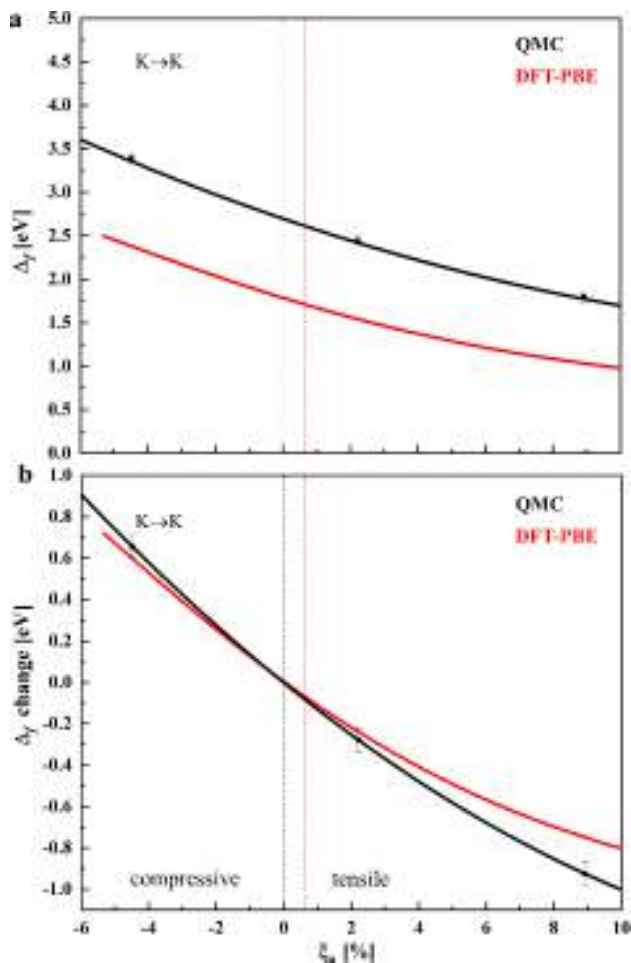
band gap phase diagram



SOC splittings:



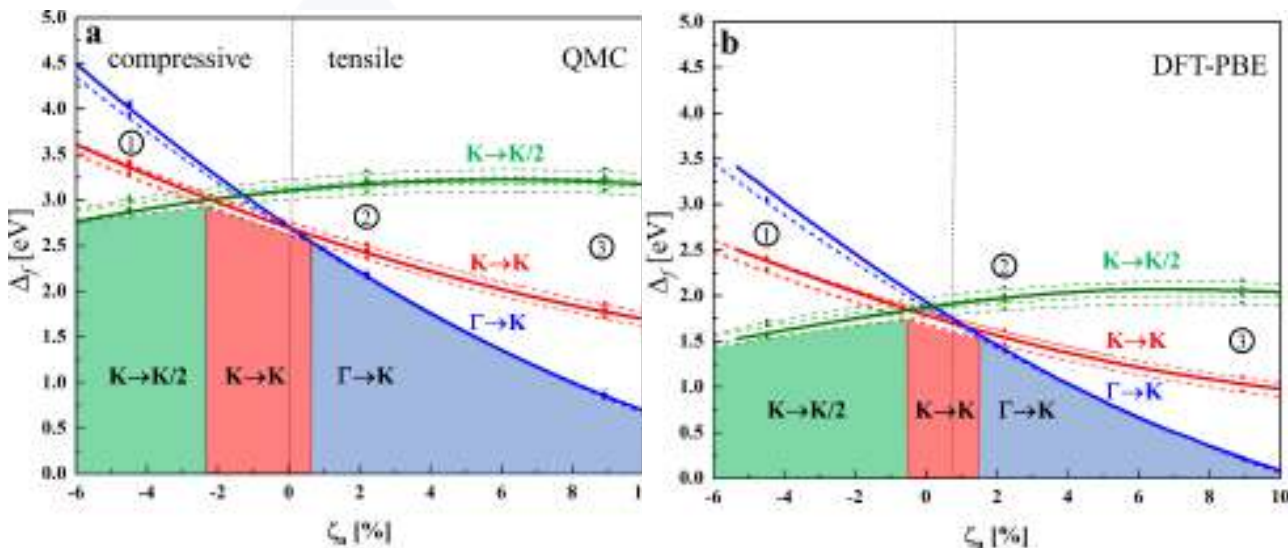
K→K excitation



DFT-PBE rigid band gap offset by ≈ 1 eV

qualitatively similar results

band gap phase diagram



transition	gauge factor					exp
	QMC	GW	PBE	HSE	B3LYP	
K→K	136/60	138/60	104/47	149	145	40-125
Γ→K	227	271	198	198	196	-
K→K/2	42	83	40	41	41	-

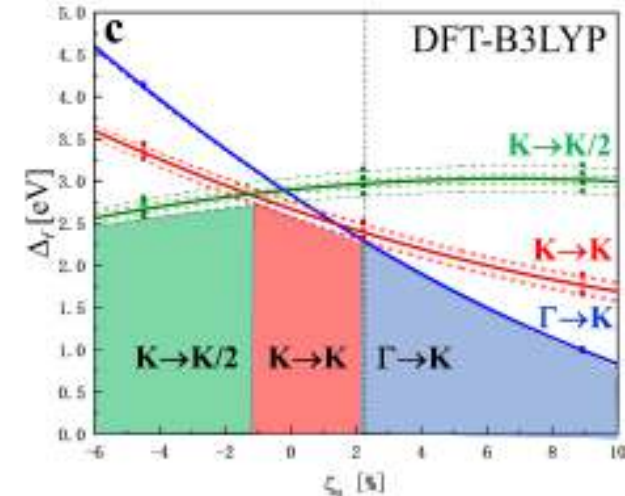
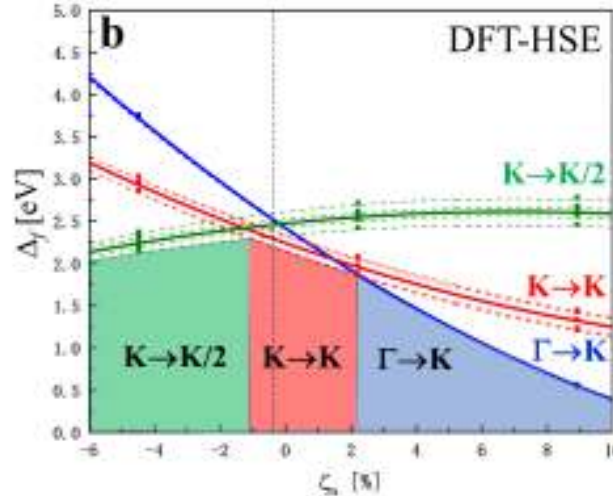
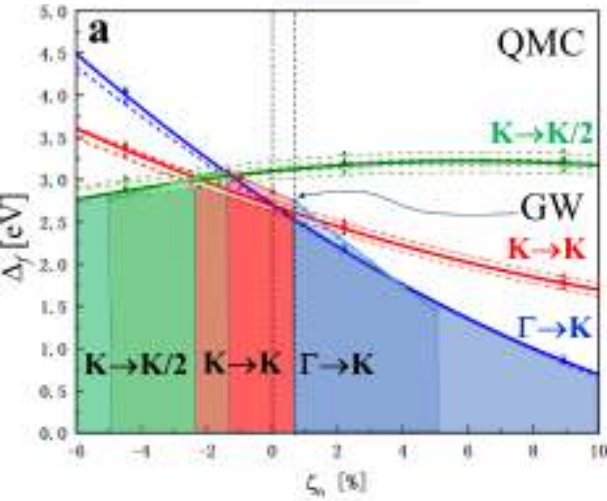
⇒ qualitatively similar results ⇒ ???coincidence or trend for ALL TMDs and TREATMENTS???

comparison of band gap phase diagrams

GW: tuning only in **compressive strain**

DFT-HSE: tuning mostly in **tensile strain**

DFT-B3LYP: tuning only in **compressive strain**



qualitatively similar phase diagrams

transition	gauge factor					exp
	QMC	GW	PBE	HSE	B3LYP	
K→K	136/60	138/60	104/47	149	145	40-125
Γ→K	227	271	198	198	196	-
K→K/2	42	83	40	41	41	-

⇒ similar gauge factors

⇒ excitations similar response to strain

⇒ excitations different values

⇒ **relation between different excitations different**

extremely sensitive indicator of el. structure quality

- ❑ excitation/neutral gap = quasiparticle gap
optical gaps in 2nd order theory
- ❑ electronic & atomic structure of 2D materials can be studied ultra-accurately with FNQMC methods
- ❑ direct band-gap tunability in tensile strain
- ❑ quintessential straintronic material MoS₂ tunability 10× smaller than in phosphorene
- ❑ tunability of phosphorene is colossal