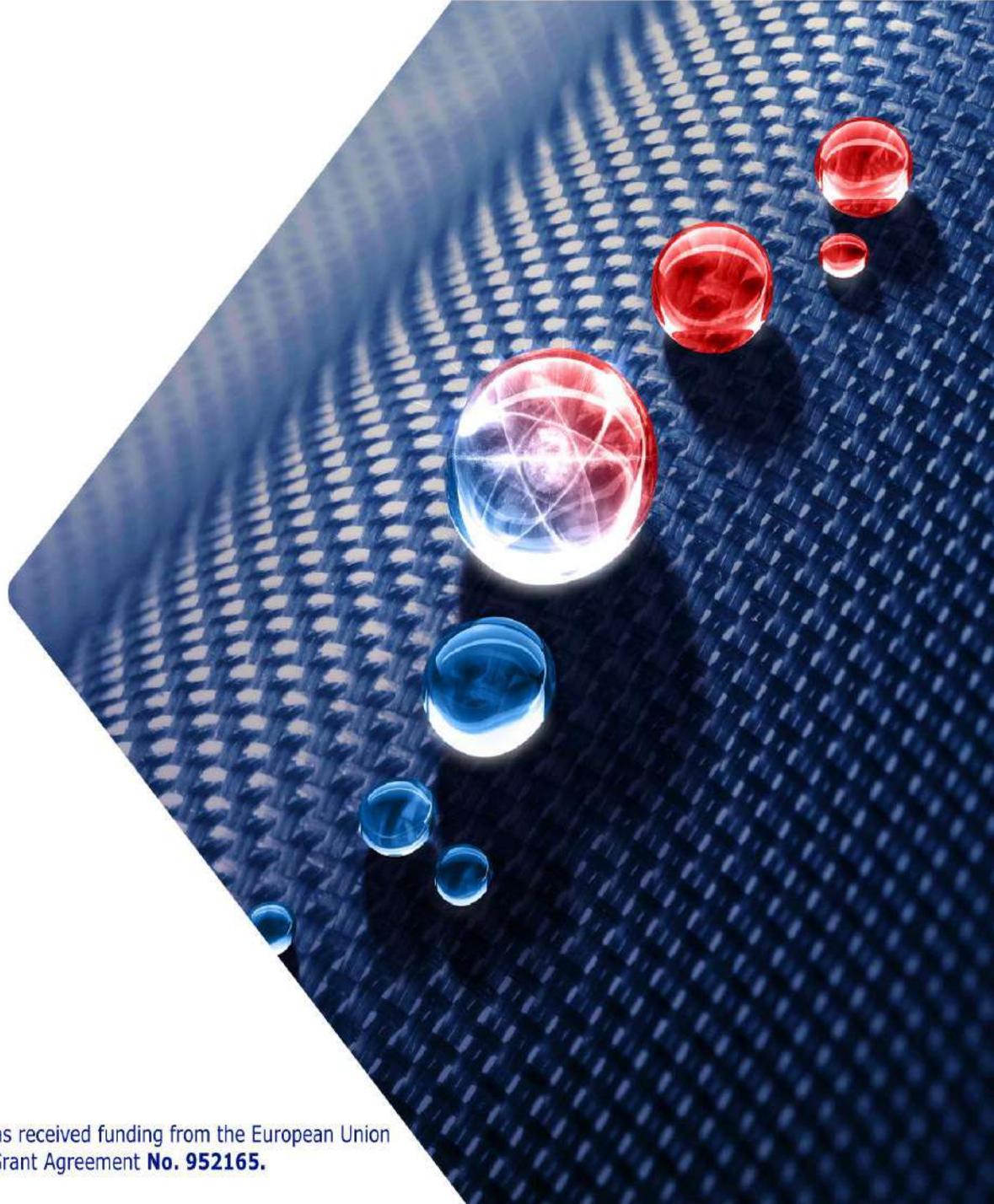




Targeting Real chemical accuracy at the EXascale

## Quantum Monte Carlo study of straintronic response of 2D materials: monolayer phosphorene and MoS<sub>2</sub>

**I. Stich**  
**Inst. of Informatics,**  
**Slovak Academy of Sciences**



Targeting Real Chemical Accuracy at the Exascale project has received funding from the European Union Horizon 2020 research and innovation programme under Grant Agreement **No. 952165**.

**Y. Huang, J. Brndiar, M. Manzoor, I. Štich  
L. Mitas, J. Fabian**

**CCMS, Inst. of Informatics, Slovak Academy of Sciences  
Dept. of Physics, North Carolina State University, Raleigh, U.S.A.  
University of Regensburg, Inst. for Theoretical Physics, Germany**

**SLOVAK ACADEMY OF SCIENCES**



**Funding:**

- **H2020**
- **APVV-21-0272**
- **VEGA 2/0070/21**
- **PRACE**
- **EuroHPC**



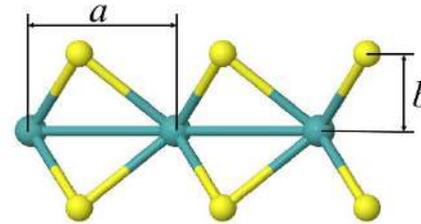
Call 24, Tier0:  
project no.  
2021250026



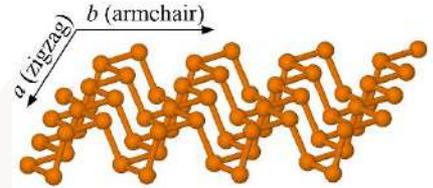
□ Intro. 2D materials

quintessential  
straintronic material

**MoS<sub>2</sub>**

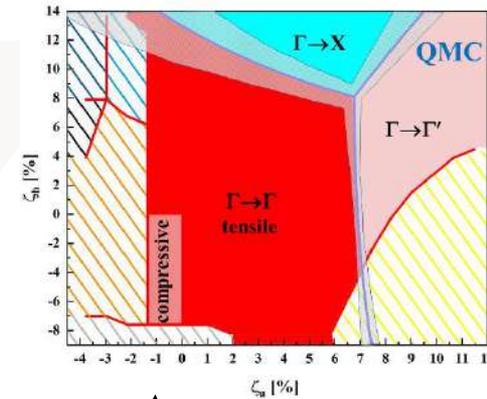
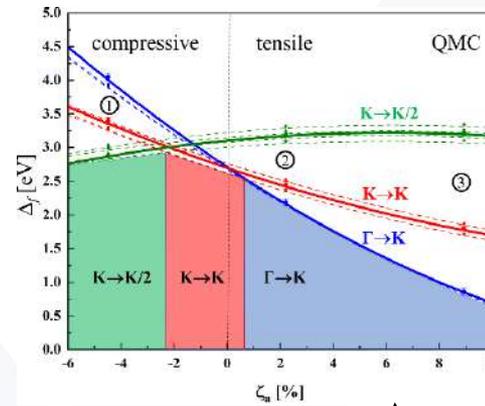


**phosphorene**



- Calculations: 1) band gap  
2) strain effects  
3) deformation energy

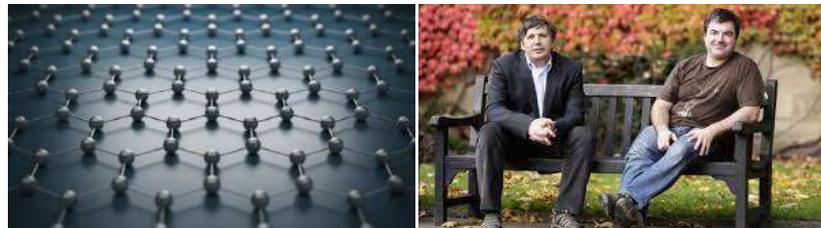
„phase diagrams“ of excitations



□ Summary

□ a number of 2D materials prepared:

- **exfoliation**
- **epitaxial growth**



**Nobel prize for physics (2010)**  
**Geim & Novoselov**

□ electronic properties:

metals  $\Leftrightarrow$  semimetal  $\Leftrightarrow$  **semiconductor**  $\Leftrightarrow$  **wide gap insulator**

Pd, Rh      graphene

**TMD**  
**phosphorene**

**h-BN**

□ high carrier mobility

□ often direct gap

2D materials highly sensitive to **strain**

2D materials can **sustain strains** well in excess of **10%**: graphene  $\approx 25\%$  and  $\text{MoS}_2 \approx 11\%$

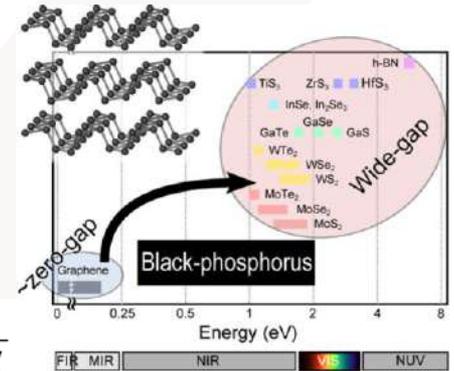
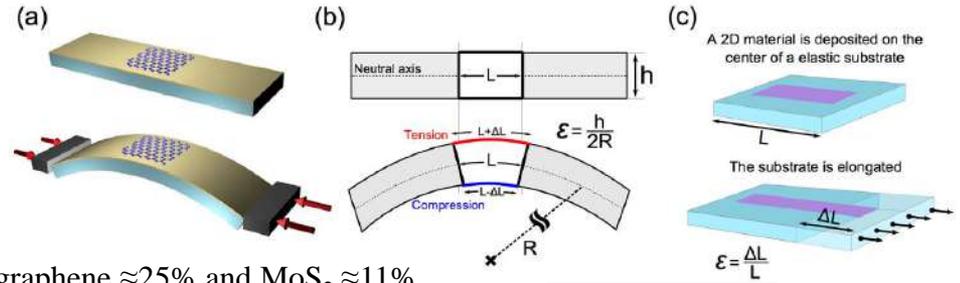
R. Roldán, A. Castellanos-Gomez, E. Capelluti, and F. Guinea, *J. Phys.: Condens. Matter* **27**, 313201 (2015)

**tuning properties** via **strain - band gap** for given applications

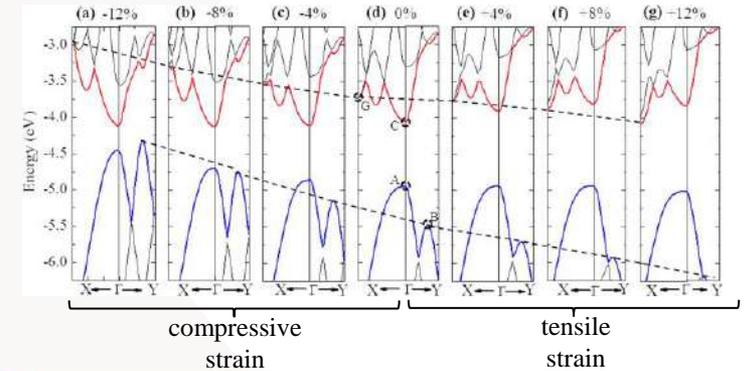
Castellanos-Gomez. A., *J. Phys. Chem. Lett.* **6**, 4280 (2015)

- **effective mass**  $\Rightarrow$  **carrier mobility**

Y. S., Y. Chen, and C. Jiang, *InfoMat.* **3**, 397420 (2021)

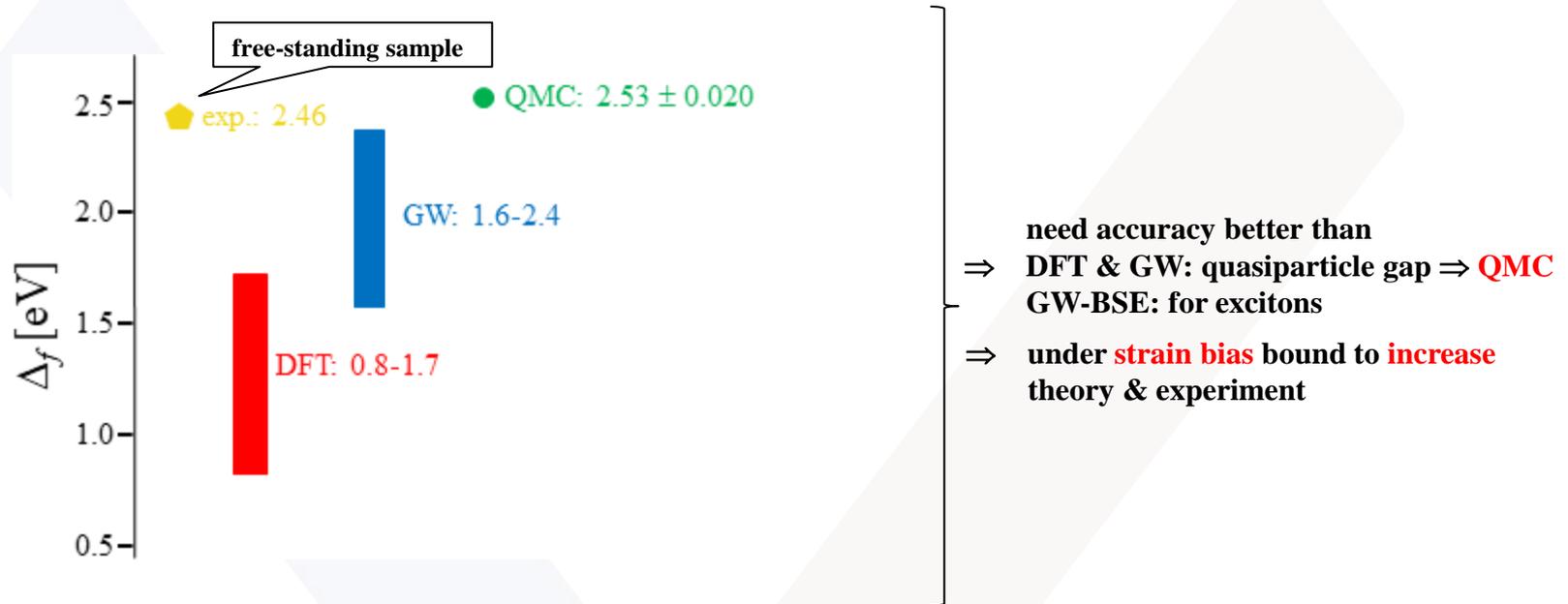


$$m^* = \frac{1}{\frac{1}{\hbar^2} \frac{d^2 E}{dk^2}}$$



**accuracy: quasiparticle band gap**

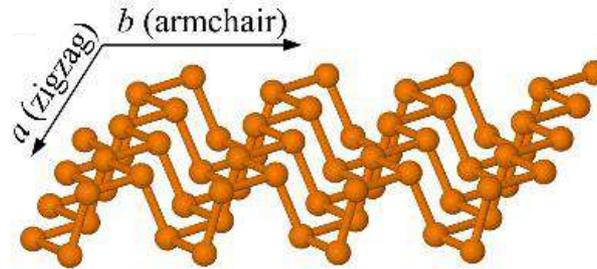
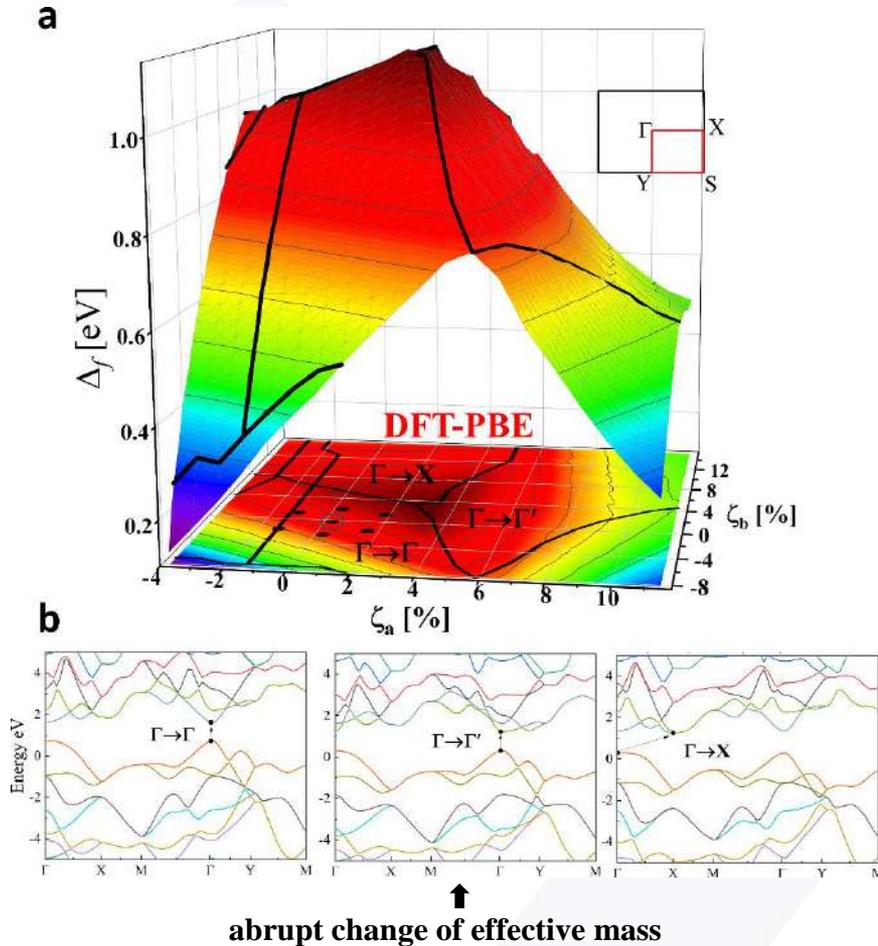
**example: monolayer phosphorene**



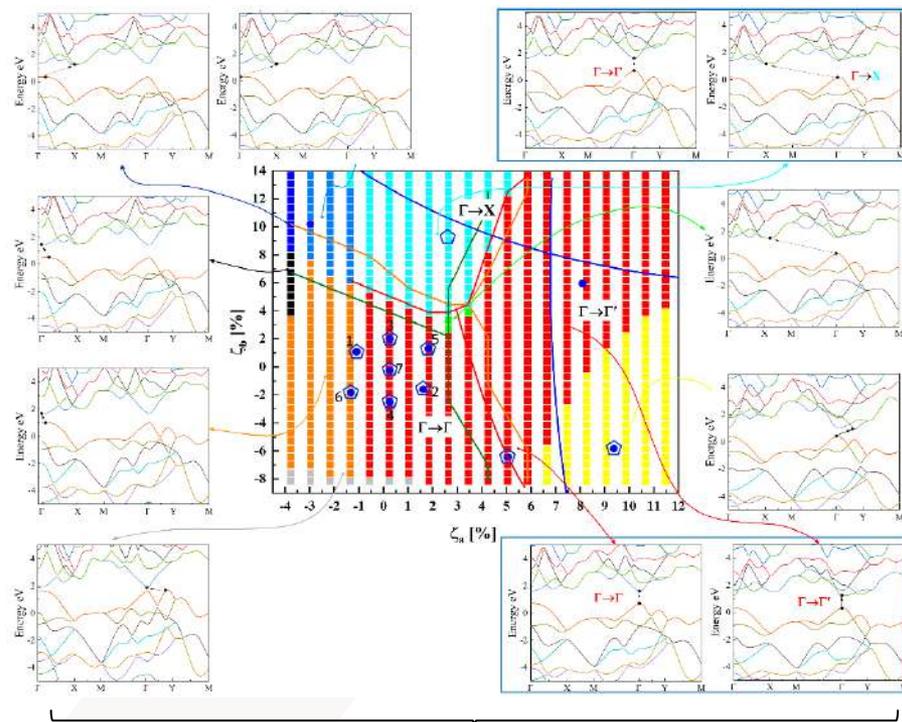
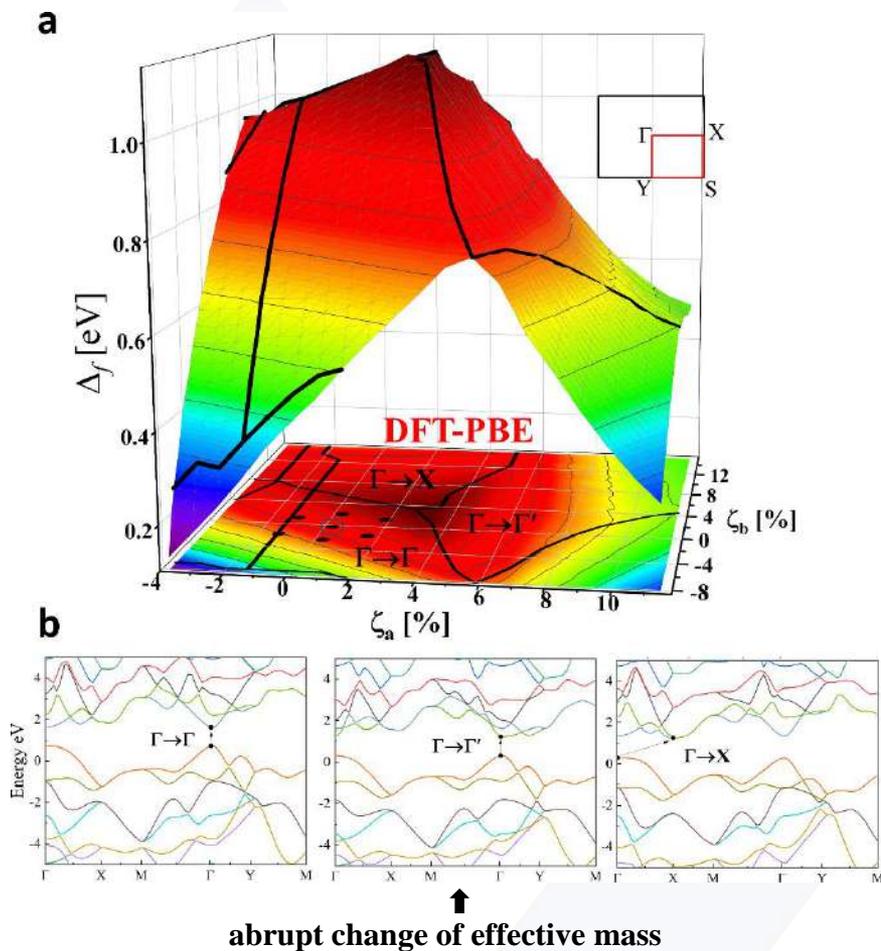
E. Gaufrès, F. Fossard, V. Gosselin, L. Sponza, F. Ducastelle, Z. Li, S. G. Louie, R. Martel, M. Côté, and A. Loiseau, Nano Lett. 19, 8303 (2019).

T. Frank, R. Derian, K. Tokar, L. Mitas, J. Fabian, and I. Stich, Phys. Rev. X 9, 011018 (2019).

qualitative picture: pre-screened by DFT-PBE DFT

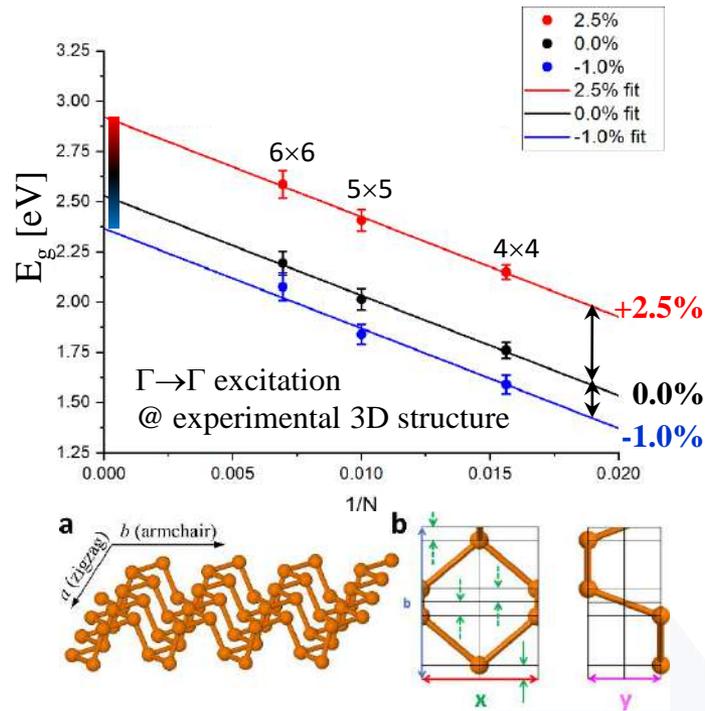


qualitative picture: pre-screened by DFT-PBE DFT



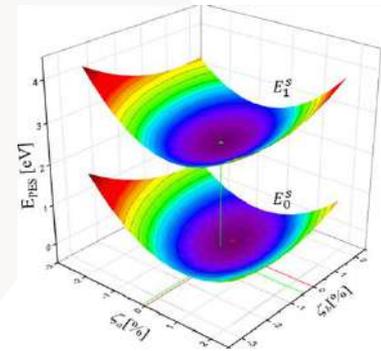
plethora of other excitations

## quantitative picture: FNQMC (with DFT-PBE nodes)



- the strain band gap **tuning** is quite **strong**
- quasiparticle band gap **scales linearly** with the applied **strain**
- **highly nontrivial** as the band gaps are differences between ground- and excited-state PESs.

converged ground- and excited-state **PESs**  
 parallel; only vertical & lateral offsets

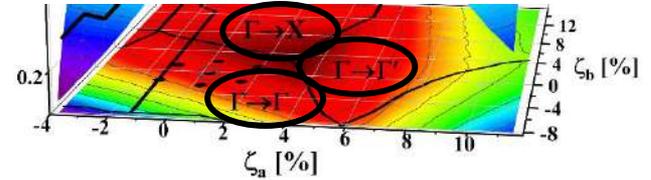


$\Rightarrow$  4 parameters  $a$ ,  $b$ ,  $x$ , and  $y$

bracketted by at least 25 QMC points

fixing all 4 parameters  $a$ ,  $b$ ,  $x$ , and  $y$ : **quadrivariate paraboloid function**  $E_0(a, b, x, y)$ , minimize w.r.t.  $x$ , and  $y$  for each  $(a, b) \Rightarrow E_0(a, b)$   
 excited state  $E_1$  only computed at the minimum w.r.t.  $x$ , and  $y \Rightarrow E_1(a, b)$

structural approximants & finite size scaling:  
**7 points** bracketing the minimum



$\Gamma \rightarrow \Gamma / \Gamma \rightarrow \Gamma'$ :

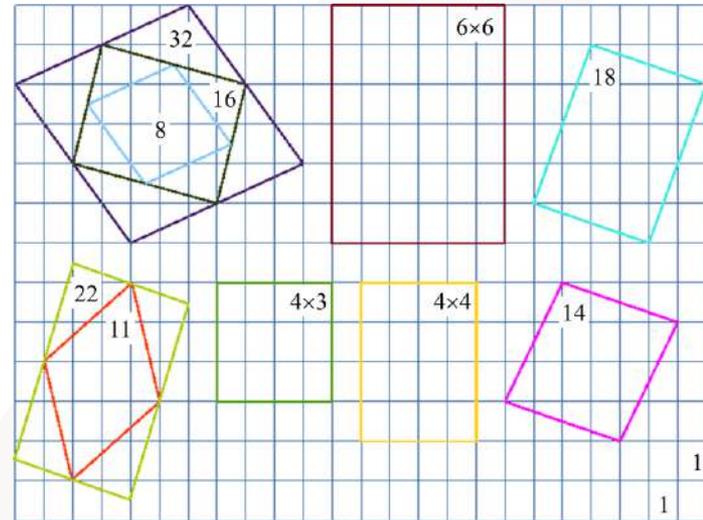
approximants:

- 11 prim. u.c. | 4D**
- 16 prim. u.c. | 4D**
- 18 prim. u.c. | 4D**
- 22 prim. u.c. | 4D**

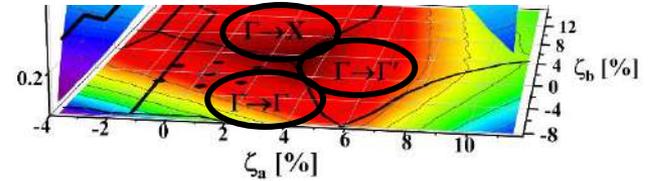
$\Gamma \rightarrow X$ :

approximants:

- 14 prim. u.c. | 4D**
- 16 prim. u.c. | 4D**
- 18 prim. u.c. | 4D**
- 22 prim. u.c. | 4D**



structural approximants & finite size scaling:  
**7 points** bracketing the minimum



$\Gamma \rightarrow \Gamma / \Gamma \rightarrow \Gamma'$ :

approximants:

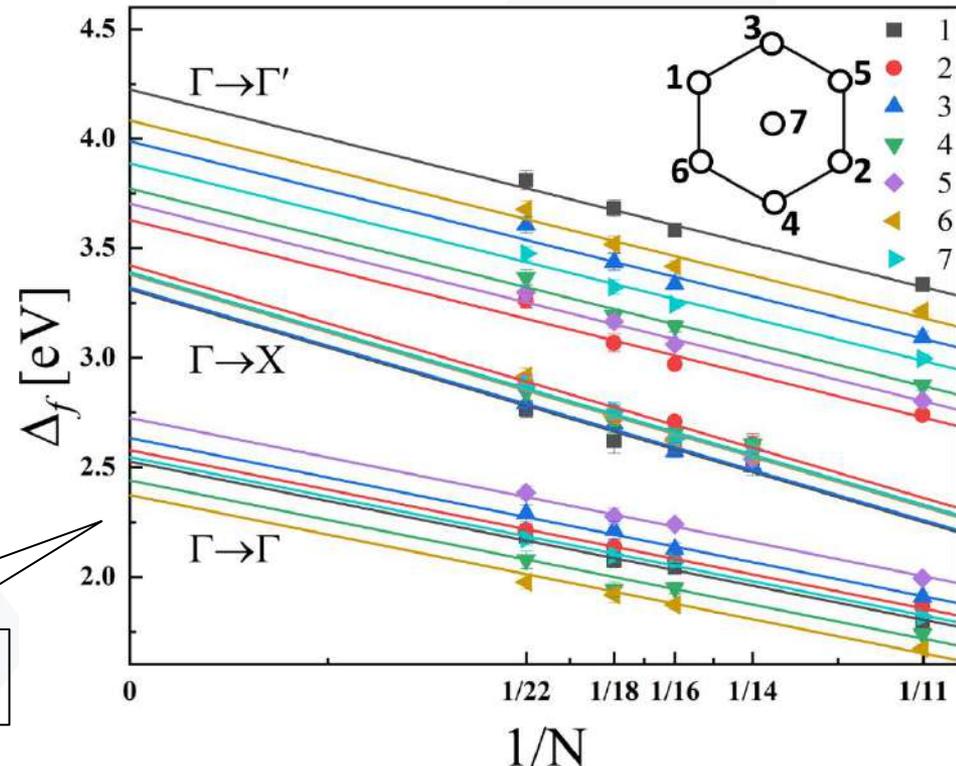
- 11 prim. u.c. | 4D
- 16 prim. u.c. | 4D
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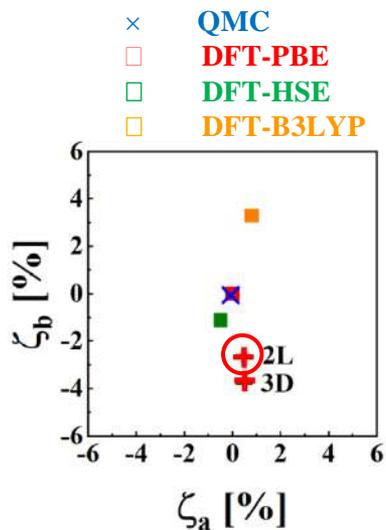
$\Gamma \rightarrow X$ :

approximants:

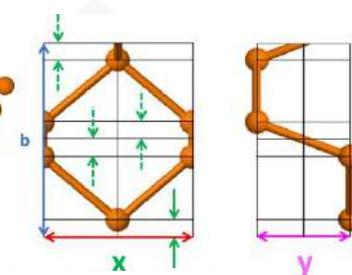
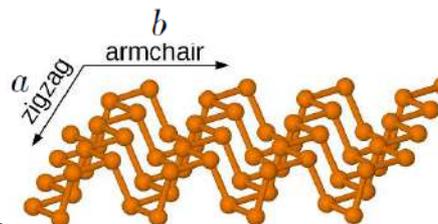
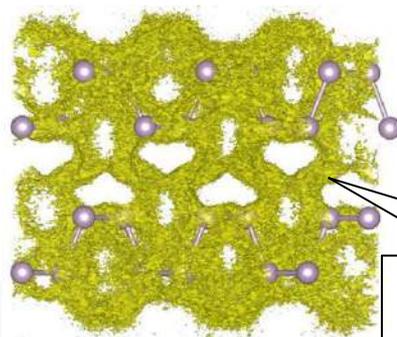
- 14 prim. u.c. | 4D
- 16 prim. u.c. | 4D
- 18 prim. u.c. | 4D
- 22 prim. u.c. | 4D

**!no twist averaging possible!**





2-layer phosphorene



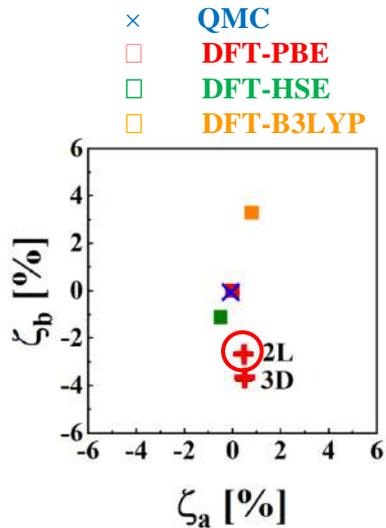
partially chemical interlayer bond  
(between lone pairs on P's)

expt.

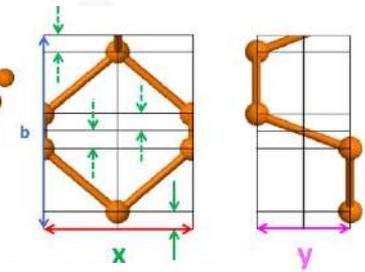
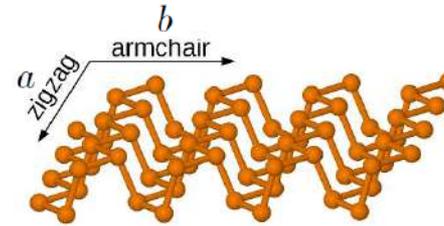
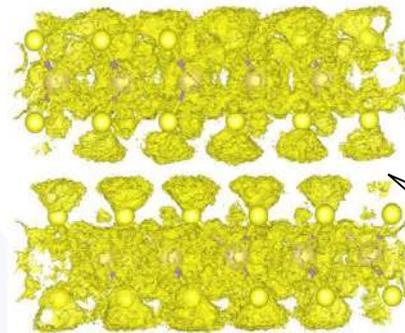
|               | QMC 11-cell    | QMC 16-cell     | QMC 22-cell     | DFT-PBE  | 3D-BP  |
|---------------|----------------|-----------------|-----------------|----------|--------|
| ground state  |                |                 |                 |          |        |
| <i>a</i>      | 6.229±0.008    | 6.238±0.002     | 6.230±0.002     | 6.235    | 6.2618 |
| <i>b</i>      | 8.651±0.017    | 8.688±0.008     | 8.707±0.006     | 8.711    | 8.2700 |
| <i>x</i>      | 0.748±0.005    | 0.759±0.002     | 0.765±0.001     | 0.770    | 0.6367 |
| <i>y</i>      | 3.987±0.007    | 3.978±0.003     | 3.985±0.002     | 3.975    | 4.0280 |
| $E_0^*$       | -716.563±0.002 | -716.498±0.0008 | -716.469±0.0007 | -717.711 | NA     |
| excited state |                |                 |                 |          |        |
| <i>a</i>      | 6.227±0.027    | 6.222±0.003     | 6.224±0.003     | 6.222    | NA     |
| <i>b</i>      | 8.515±0.178    | 8.607±0.012     | 8.664±0.010     | 8.543    | NA     |
| <i>x</i>      | 0.728±0.044    | 0.747±0.003     | 0.761±0.002     | 0.739    | NA     |
| <i>y</i>      | 4.053±0.018    | 4.031±0.002     | 4.020±0.002     | 4.051    | NA     |
| $E_1^*$       | -716.415±0.005 | -716.375±0.0007 | -716.375±0.001  | -717.643 | NA     |

$$\begin{aligned}
 a &= 3.30 \pm 0.003 \text{ \AA} \\
 b &= 4.61 \pm 0.006 \text{ \AA} \\
 x &= 0.405 \pm 0.001 \text{ \AA} \\
 y &= 2.109 \pm 0.001 \text{ \AA}
 \end{aligned}$$

not known experimentally



2-layer MoS<sub>2</sub>



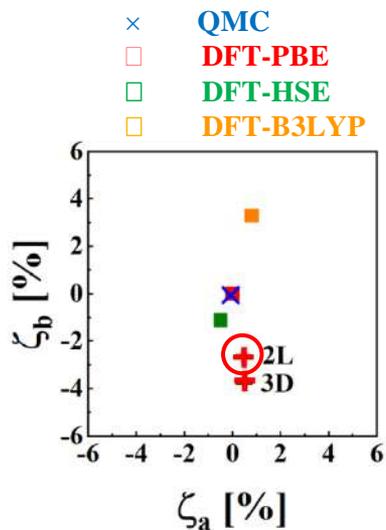
pure vdW interaction

expt.

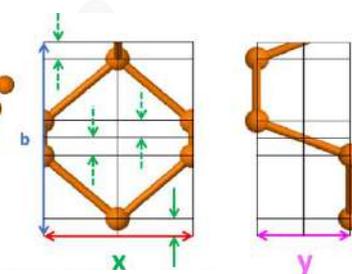
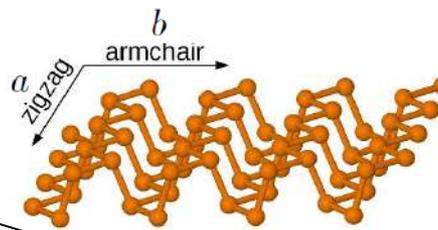
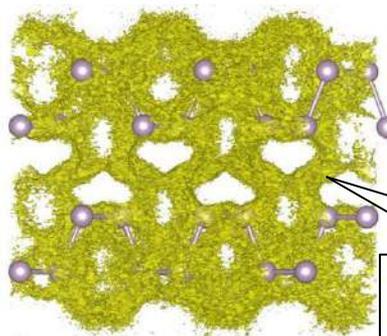
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| <i>y</i>                         | 4.053±0.018    | 4.031±0.002     | 4.020±0.002     | 4.051    | NA     |
| <i>E<sub>1</sub><sup>*</sup></i> | -716.415±0.005 | -716.375±0.0007 | -716.375±0.001  | -717.643 | NA     |

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 \end{aligned}$$

not known experimentally



2-layer phosphorene



partially chemical interlayer bond  
(between lone pairs on P's)

expt.

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 \end{aligned}$$

not known experimentally

$$\Delta_f = 2.53 \pm 0.020 \text{ eV} \Leftrightarrow 2.46 \text{ eV}$$

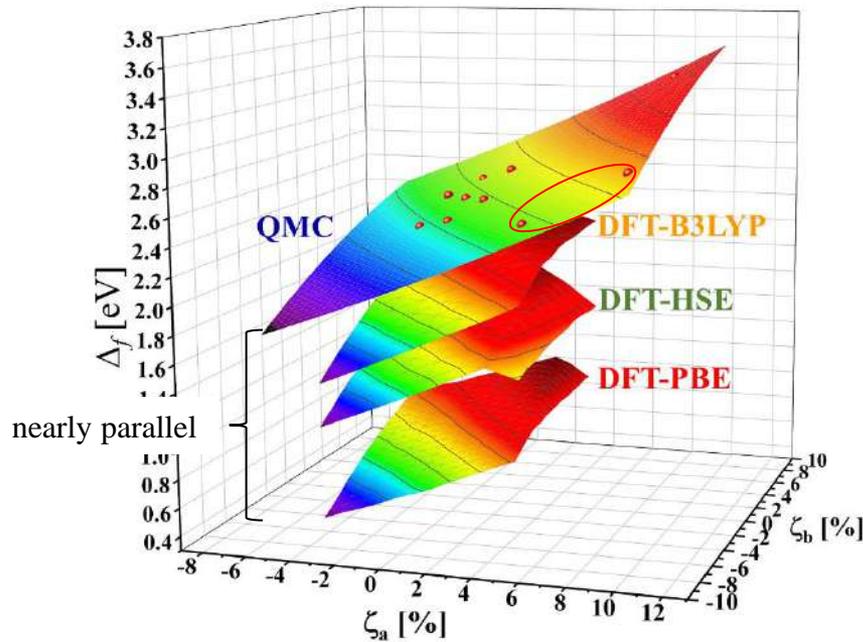
experimental value for freestanding phosphorene

↑  
 (almost) **chemical precision**  
 (no vibronic and zero-point corrections)

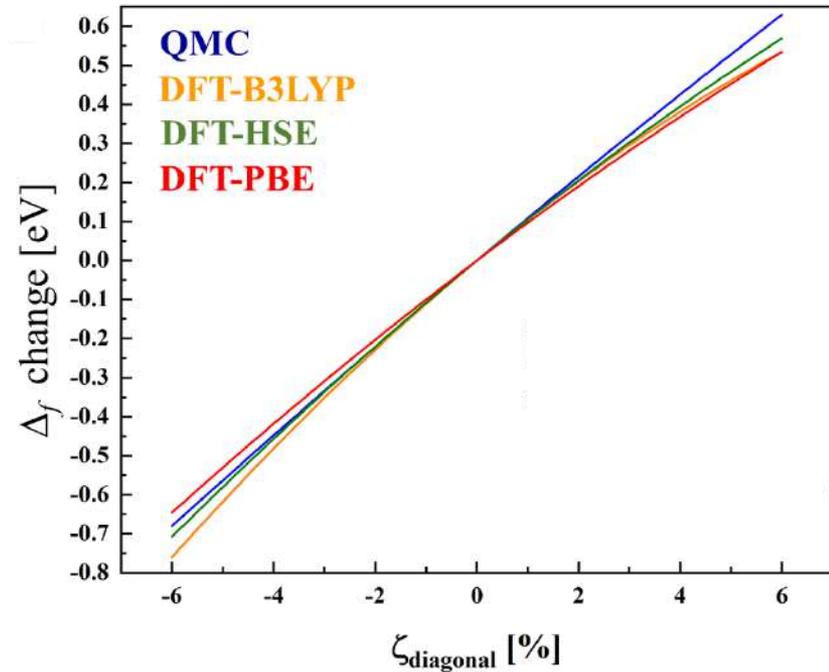
band gap

$\Gamma \rightarrow \Gamma$  excitation

gauge factor



**QMC** consistently gives **larger gap** than any common DFT functional (**PBE**, **HSE**, **B3LYP**),  
**!ζ = 0 agrees with experiment!**



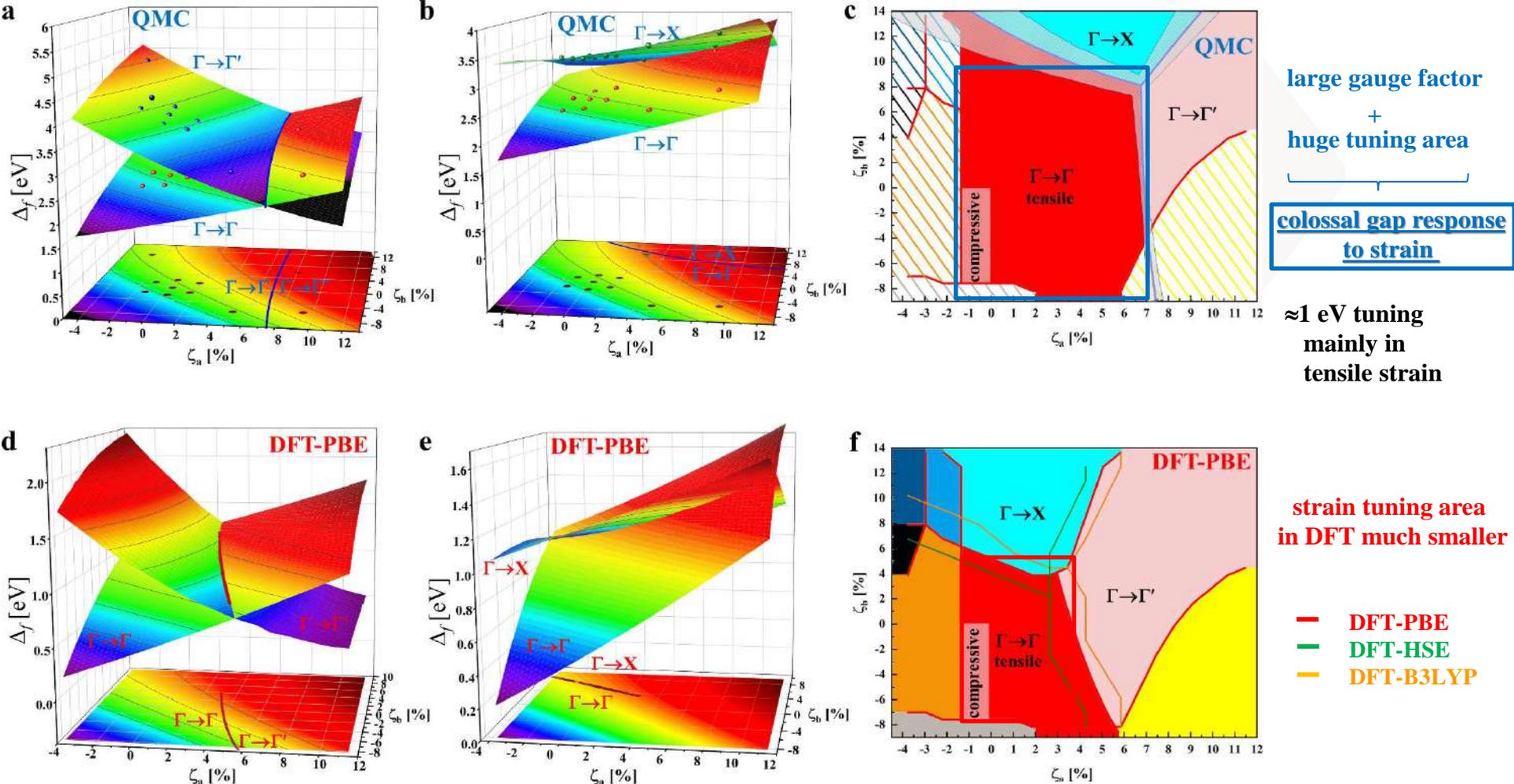
**gauge factor** (change of  $\Delta_f$ ) very **similar to DFT**  
 (if computed against the respective minimum)

**large gauge factor:  $\approx 100$  meV/%**

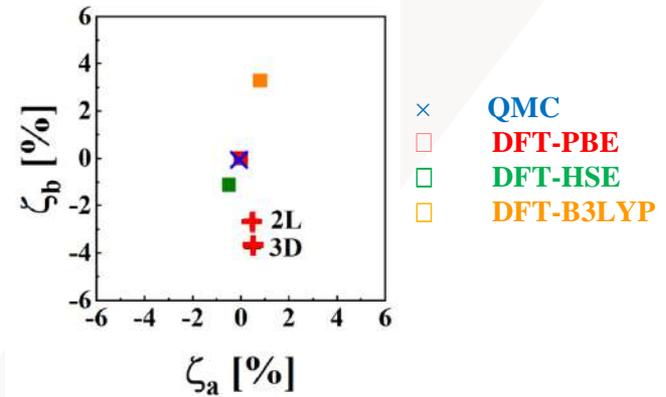
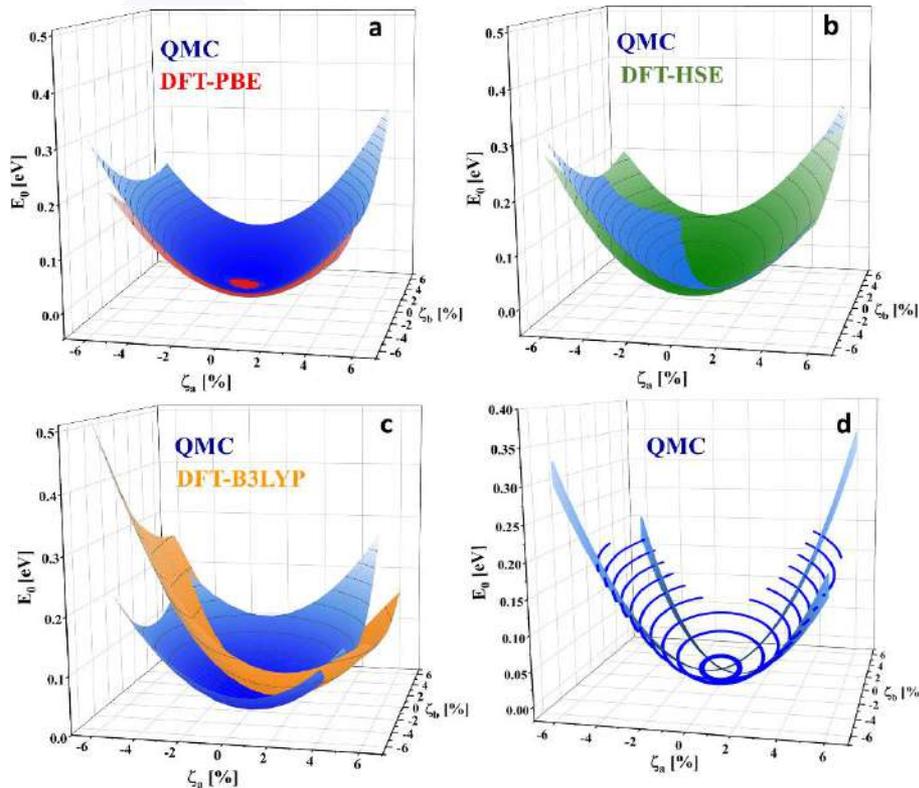


**main straintronic materials  $\text{MoS}_2$ :  $64 \pm 5 / 68 \pm 5$  meV/%**

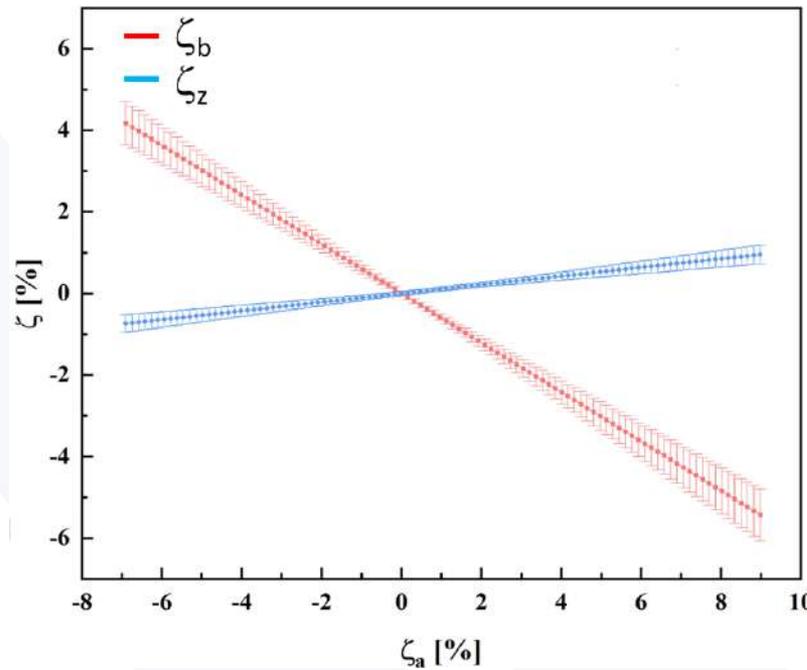
quantitative picture: **QMC**  $\Gamma \rightarrow \Gamma$   $\Gamma \rightarrow \Gamma'$   $\Gamma \rightarrow X$  boundaries  $\Rightarrow$  band gap phase diagrams



Y. Huang, A. Faizan, M. Manzoor, J. Brndiar, L. Mitas, J. Fabian, and I. Štich, Phys. Rev. Research 5, 033223 (2023).



## Poisson's ratio

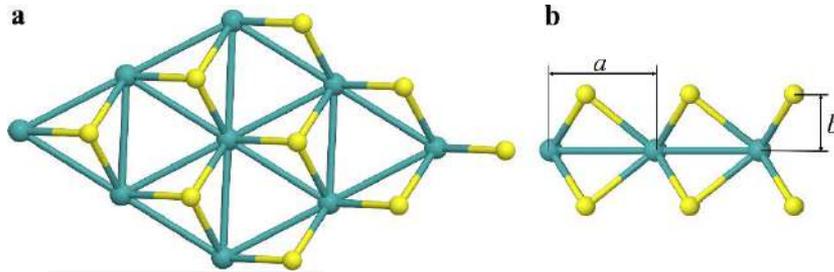


positive in-plane

negative out-of-plane

Poisson's ratio

auxetic material



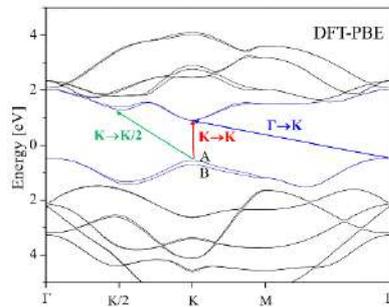
consider only diagonal strain ( $a$ )

$\Rightarrow$  2 parameters  $a, b$

fixing 2 parameters  $a, b$ : **bivariate paraboloid function**  $E_0(a, b)$ , minimize w.r.t.  $b$  for each  $a \Rightarrow E_0(a)$

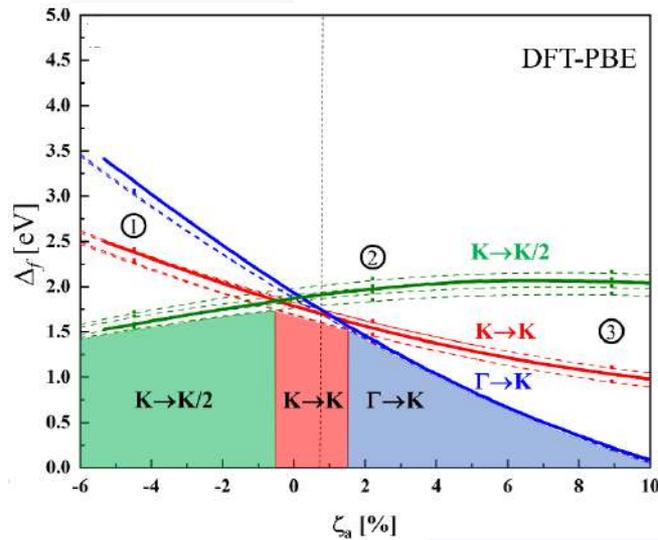
excited state  $E_I$  only computed at the minimum w.r.t.  $b \Rightarrow E_I(a)$

spin-orbit couplings (SOC) small but non-negligible  $\Rightarrow$  **SOC** added a posteriori **perturbatively** based on **DFT-PBE**

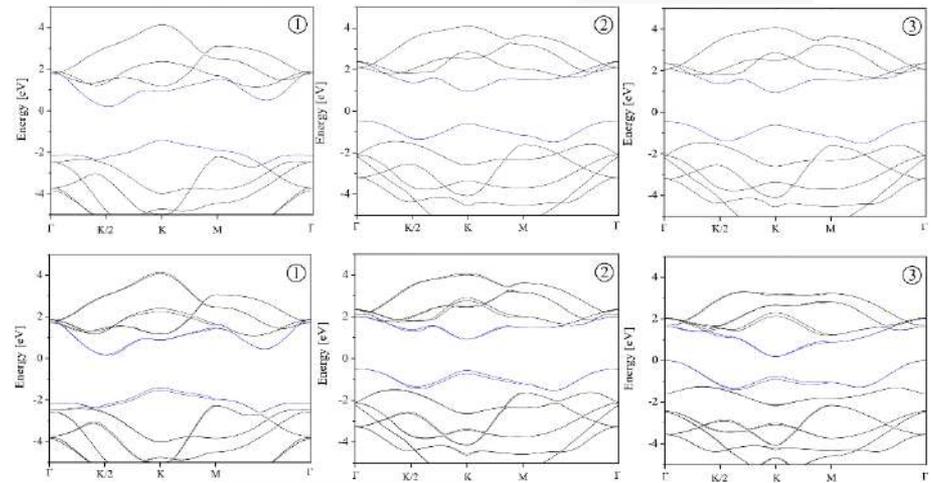


qualitative picture: pre-screened by DFT-PBE DFT

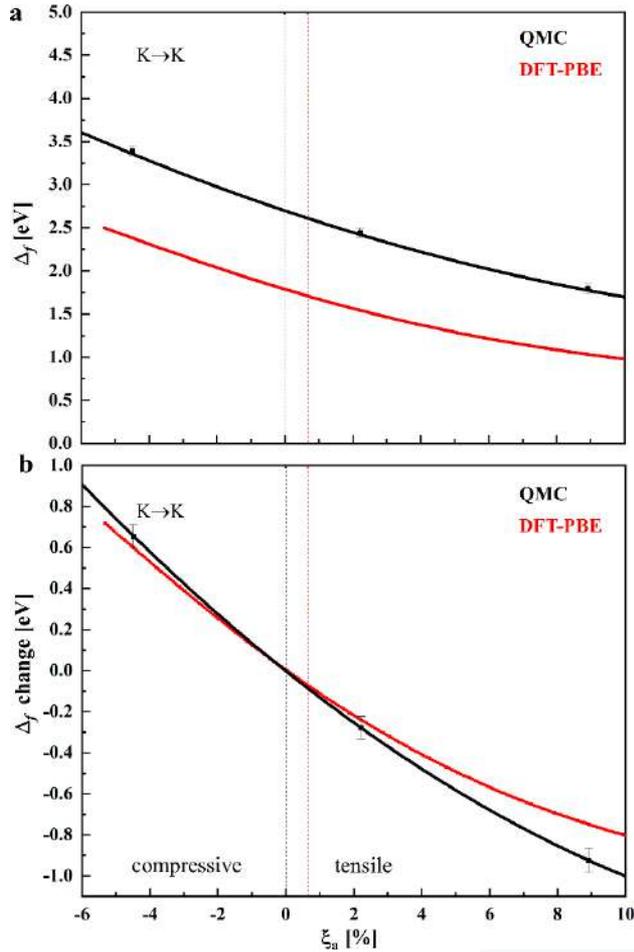
band gap phase diagram



SOC splittings:



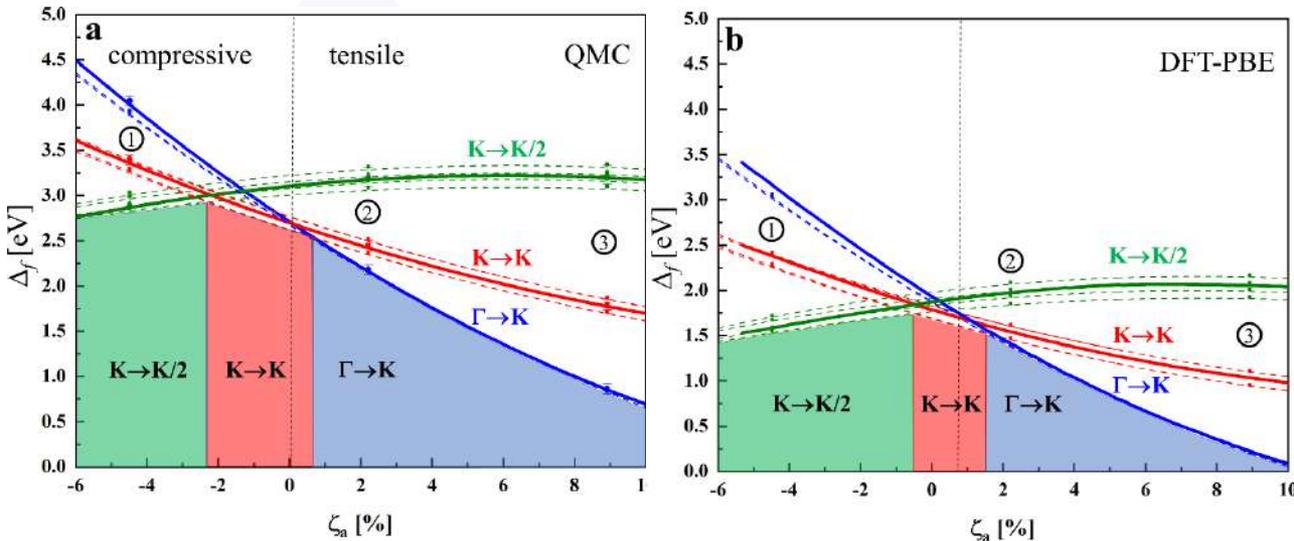
## K→K excitation



DFT-PBE rigid band gap offset by  $\approx 1$  eV

qualitatively similar results

band gap phase diagram



| transition | gauge factor |        |        |     |       | exp    |
|------------|--------------|--------|--------|-----|-------|--------|
|            | QMC          | GW     | PBE    | HSE | B3LYP |        |
| K→K        | 136/60       | 138/60 | 104/47 | 149 | 145   | 40-125 |
| Γ→K        | 227          | 271    | 198    | 198 | 196   | -      |
| K→K/2      | 42           | 83     | 40     | 41  | 41    | -      |

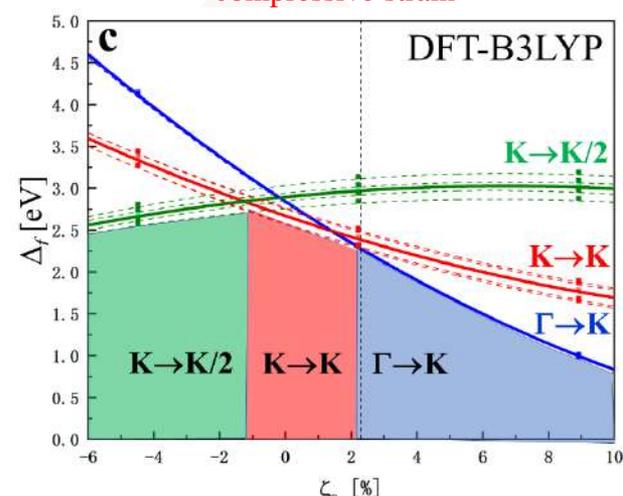
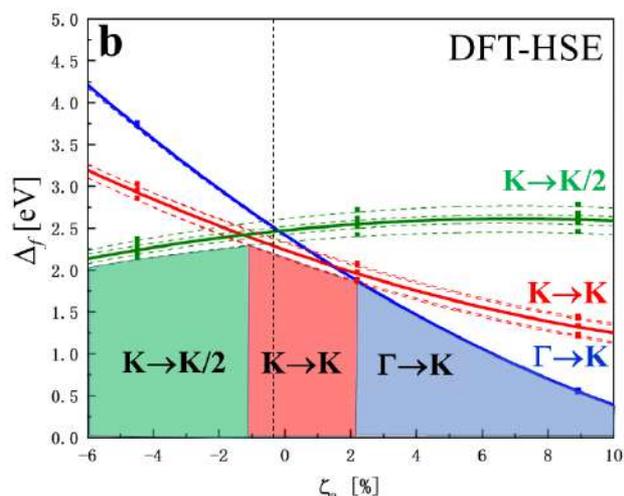
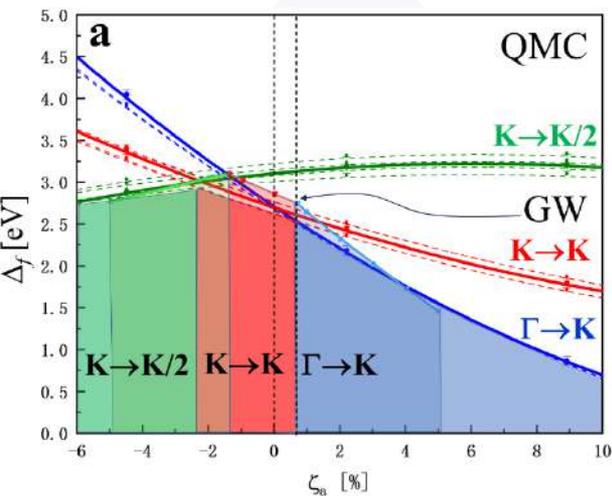
⇒ qualitatively similar results    ⇒ ???coincidence or trend for ALL TMDs and TREATMENTS???

comparison of band gap phase diagrams

GW: tuning only in **compressive strain**

DFT-HSE: tuning mostly in **tensile strain**

DFT-B3LYP: tuning only in **compressive strain**



qualitatively similar phase diagrams

| transition | gauge factor |        |        |     |       | exp    |
|------------|--------------|--------|--------|-----|-------|--------|
|            | QMC          | GW     | PBE    | HSE | B3LYP |        |
| K→K        | 136/60       | 138/60 | 104/47 | 149 | 145   | 40-125 |
| Γ→K        | 227          | 271    | 198    | 198 | 196   | -      |
| K→K/2      | 42           | 83     | 40     | 41  | 41    | -      |

⇒ similar gauge factors

⇒ excitations similar response to strain

⇒ excitations different values

⇒ relation between different excitations different

extremely sensitive indicator of el. structure quality

- ❑ electronic & atomic structure of 2D materials can be studied ultra-accurately with FNQMC methods
- ❑ direct band-gap tunability in tensile strain
- ❑ quintessential straintronic material  $\text{MoS}_2$  tunability  $10\times$  smaller than in phosphorene
- ❑ tunability of phosphorene is colossal