

# A Jastrow wave function for the spin-1 Heisenberg chain: string order revealed by the mapping to the classical Coulomb gas

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We show that a two-body Jastrow wave function is able to capture the ground-state properties of the  $S = 1$  antiferromagnetic Heisenberg chain with the single-ion anisotropy term, in both topological and trivial phases [1]. Inspecting the Jastrow wave function, optimized using the stochastic reconfiguration technique [2], we recognize that its pseudo-potential assumes a very simple form in Fourier space, i.e.,  $v_q \approx 1/q^2$ , which is able to give rise to a finite string-order parameter in the topological regime. The results are analysed by using an exact mapping [3] from the quantum expectation values over the variational state to the classical partition function of the one-dimensional Coulomb gas of particles with charge  $q = \pm 1$ . Here, two phases are present at low temperatures: the first one is a diluted gas of dipoles (bound states of particles with opposite charges), which are randomly oriented (describing the trivial phase); the other one is a dense liquid of dipoles, which are aligned thanks to the residual dipole-dipole interactions (describing the topological phase, with the finite string order being related to the dipole alignment). Our results provide an insightful interpretation of the ground-state nature of the spin-1 antiferromagnetic Heisenberg model.

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[3] M. Capello, F. Becca, S. Yunoki, and S. Sorella, Phys. Rev. B **73**, 245116 (2006).